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Abstract

Developing countries have a vast informal sector generally associated with low levels of productivity. The persistence of informality could be a response to rigidities in the labor market, associated with a combination of high non-wages cost and high minimum wages. This paper proposes a theoretical framework to understand the tax policies' role that discourages informality, such as lower payroll taxes in the formal sector or increases enforcement expenditure in an economy with real wage rigidities. I develop a search and matching model with a shirking mechanism with formal and informal workers. The simulations results suggest that the magnitud effect of tax polcies depends on real wage rigidities. In relative terms, when the economy has high real wage rigidities, the reduction of payroll taxes has a greater effect reducing the informality. In contrast, when the economy has low real wage rigidities, the enforcement expenditure has a significant effect to reduce the informality. Also, the results show the existence of tax polcies combition that reduce the informal labor in an effective way

Keywords: Informality, Tax policies, Enforcement expenditure, Fiscal policies, Search and matching, Efficiency wage, Shirking mechanism.

JEL classification: JEL J46, E26, E62, O17, H26

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1 Introduction

The informal sector, defined as a labor force that does not comply with government regulations, has a remarkable size that persists over time. Specifically, developing countries have a vast informal sector generally associated with low productivity levels (Perry, 2010). A high share of informality is related to less pension and health cover, workers without unemployment compensation, and less tax revenue via widespread tax evasion (Schneider & Enste, 2000). Also, the persistence of informality could be a response to rigidities in the labor market, associated with a combination of high non-wages cost and high minimum wages (Maloney, 2004). The informal sector in Colombia is one of the most significant issues, is associated with low productivity and with the vulnerability of the long-run fiscal sustainability.Some authors as (Mondragón-Vélez et al., 2010; Santa María et al., 2010) highlight the relevance of the labor market rigidities to understand the persistence of the informality.

Accordingly, this paper sheds ligth to understand the effect of policies that seek to reduce the informality in an case with real wage rigidities and restrictions associated with the search and matching process in the labor market. In specific, The model proposed in this paper is similar in spirit to (Martin & Wang, 2020) but with a government in a dual labor market with informal and formal workers. The model seeks to understand how the combination of labor policies and the state capacity could decrease informal workers, increase formal workers, and increase government income in an economy with real wage rigidities in the formal sector.

The model has formal and informal firms that are hiring formal and informal workers, respectively. The formal firm can hire a formal workers by a matching process as (Pissarides, 2000). In contrast, the informal firm does not have any friction and hire informal workers given the marginal labor productivity. The unemployed in the economy search a job in the formal or informal sector, if the unemployed is employed in the formal firm they could be shirking or non-shirking workers as in (Shapiro & Stiglitz, 1984)

The real wage rigitidites in the formal sector comes from shirking worker throught shirking mechanim. That is the workers has am additional disutility to become non-shikirng workers in terms of real wage. The real wage increase as a consequence of the incentive from the firm to hire non-shirking worker leading to a real wage rigidities in the economy.

The government's main objective is to increase its revenues through two tax policies: reduce payroll tax in the formal sector and increase the expenditure in law enforcement on the informal sector. The first policy decreases the cost of hiring in the formal sector, but the other increases the cost of working in the informal sector. In this sense, the mechanism of both policies is to reduce informality and increase the formal labor in the economy. Given the above, the increase in the formal worker leads to an increase in the tax base and an increase in the government income.

This paper is related to the literature exploring the effect of tax or fiscal policies in reducing the informal sector. Specifically, the paper considers the contribution of M. Gómez et al. (2021), Acosta-Henao (2020) among others, who see the impact of different fiscal policies to reduce informality. More specific is related to (Pratap & Quintin, 2006; Santa María et al., 2010; Flórez, 2015; Osorio-Copete, 2016) whose research focuses on reducing the payroll taxes to increase the formality. Also, it is related with (Meghir et al., 2015; Ulyssea, 2018; Bardey & Mejía, 2019) how to see the effect of reducing informality for policies associated with law enforcement in the informal sector.

At the rigidities in the labor market, the paper is related with (Shimer, 2005; Hall, 2005; Krause & Lubik, 2007) whose explore the different effect in the real wage rigidities on the economy. Finally, the research of this paper is related with authors as (Kugler & Kugler, 2009; Antón, 2014; H. J. Gómez et al., 2015; Kugler & Kuler, 2015; Fernández & Villar, 2017) how to improve the knowledge about how should implement the policies to reduce the informality in Colombia.

To my knowledge, there is not research that linked the effect of the tax policies on informality under a labor market with real rigidities in the wage. To understand these dynamics, I first seek to tackle the possible trade-off between reducing the tax in the formal sector or increasing law enforcement to reduce informality and boost tax collection. Second, I pretend to understand what kind of policy should be applied depending on the real wage rigidities. Third, I observe how these policies affect the unemployment level.

2 Model

I develop a dynamic general equilibrium model to understand the role of tax policies defined as taxes and regulation enforcement policies in an economy with real wage rigidities through shirking mechanisms and search and matching processes. The model is based on Martin & Wang (2020), who modified the search and matching model summarized in Pissarides (2000), incorporating a shirking mechanism by Shapiro & Stiglitz (1984). Nevertheless, I include an informal sector in which there are not labor market frictions.

The model has three agents: households, firms, and government. The representative household supplies labor owns firms in the economy and chooses consumption to maximize an intertemporal function. The workers in the economy belong to one of three states: informal worker, formal worker or unemployed in the formal sector, employed in the informal sector, or be unemployed. If the worker is employed in the formal sector, they could be shirking or non-shirking workers.

The firms are formal or informal. Formal firms can hire up to one worker, and these workers could be shirking or non-shirking. Also, there is a fixed vacancy cost; in this sector, the formal wage is determined by shirking mechanisms. On the other hand, informal firms are labor-intensive and have marginal decreasing returns; wages in this sector correspond to workers' marginal productivity.

Finally, the government has a primary balance that depends on revenue and expenditure. The government revenue comprises formal firms' taxes, households' taxes, and informal firms' fines. In contrast, the government expenditure depends on the transfer to households, unemployment transfer, enforcement expenditure to fine informal firms, and unproductive expenditure.

Moreover, the government has two principal tax policies to increase the formal sector and government revenue: reduce the formal firms' taxes, and change the enforcement expenditure to increase the probability of auditing and fining an informal firm. The above policies create incentives to transition from the informal sector to the formal sector, increasing government revenue by increasing the taxable base.

2.1 Households

The representative households derives utility from consumption according to the intertemporal utility function.

$$\max_{c_t, b_t} E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \left(U(c_t^{wi} - \chi) l_t^i + U(c_t^{wns} - \chi - \zeta) l_t^{wns} + U(c_t^{ws} - \chi) l_t^{ws} + U(c_t^u) u_t\right) \tag{1}$$

Where ρ is the time discount rate. c_t^{wi}, c_t^{wns}, c_t^{ws}, and c_t^u are per capita consumption for informal workers, non-shirking workers, shirking workers, and unemployed respectively. For simplicity, I assume the functions $U(\cdot)$ are strictly increasing and strictly concave. Households own firms and workers in the economy, and there is risk-sharing. In each period, the household earns (and consumes) real wage from the labor supply in the formal sector and informal sector defined as w_t^f and w_t^i respectively.

Formal and informal workers suffer a disutility χ. Also, employed in the formal can be shirking or non-shirking. Hence, workers who do not shirk incur a disutility ζ. Furthermore, in each period, the household receives formal firms dividends Div_t, and an unemployment transfer for the government given by s_t. By the above, the household's problem consists of maximizing the intertemporal utility function (1) subject to the following budget constraint:

$$c_t + b_t = (1 + (1 - \tau_t^{\pi})r) b_{t-1} + w_t^f l_t^f + w_t^i l_t^i + s_t u_t + (1 - \tau_t^{\pi}) Div_t \tag{2}$$

In equation (2), the households bonds are represented by b_t, and r is the return rate of the bonds. The total household consumption c_t is defined as $c_t = c_t^{wi} l_t^i + c_t^{wns} l_t^{wns} + c_t^{ws} l_t^{ws} + c_t^u u_t$. Additionally, the government taxes the formal profits of the firms with τ_t^{π}. Given the labor market, the total labor force in the economy is divided by non-shirking workers l_t^{ns}, shirking workers l_t^s, informal workers l_t^i and unemployed u_t so in per-capita terms $1 = l_t^f + l_t^i + u_t$, where $l_t^f = l_t^{ns} + l_t^s$. Given the above, the first-order conditions from the household maximization problem is the following:

$$\frac{\partial U(c_t^{wi} - \chi)}{\partial c_t^{wi}} = \frac{\partial U(c_t^{wns} - \chi - \zeta)}{\partial c_t^{wns}} = \frac{\partial U(c_t^{ws} - \chi)}{\partial c_t^{ws}} = \frac{\partial U(c_t^u)}{\partial c_t^u} = \lambda_t \tag{3}$$

$$\lambda_t = \left(\frac{1}{1+\rho}\right) E_t \lambda_{t+1} (1 + (1 - \tau_{t+1}^{\pi}) r_{t+1}) \tag{4}$$

From equation (3) is possible to define the following consumption behavior $c_t^u = c_t^{wi} - \chi = c_t^{wns} - \chi - \zeta = c_t^{ws} - \chi$. Combining the equation (3) and (4) is possible get the following expression:

$$\frac{1}{E_t(1 + (1 - \tau_{t+1}^{\pi}) r_{t+1})} = \left(\frac{1}{1+\rho}\right) \frac{\lambda_{t+1}}{\lambda_t} = \left(\frac{1}{1+\rho}\right) \Omega_{t+1} \tag{5}$$

2.2 Firms

The firms are formal or informal. Formal firms hire up to one formal worker, and informal firms hire up to one informal worker. The formal and informal firms are homogeneous. In the formal sector, the firms maximize profits. The firms post a vacancy and unemployed search for a job. The formal wage is given by the shirking mechanism, creating the real wage rigidities. In contrast, the informal firms do not face any friction, only hire non-shirking workers and maximize profit. The marginal productivity gives the worker's wage.

Matching Process

The matching function that determines the aggregate hiring in the formal sector is the following.

$$m(u_t, v_t) = k u_t^{\phi} v_t^{1-\phi} \tag{6}$$

Where $m(u_t, v_t)$ is the number of workers hired in the formal sector, u_t is the unemployment rate, and v_t is the formal vacancy rate. As equal to the standard literature of search and matching models, ϕ and k are parameters characterizing the constant returns of the matching function. Moreover, I define the labor market tightness θ_t as the ratio of vacancy rate to the unemployment rate.

$$\theta_t = \frac{v_t}{u_t} \tag{7}$$

when θ_t is higher, the labor market is tighter from an entrepreneur perspective. It is possible to define the probability of filling a vacancy in the formal sector as $q(\theta_t)$ equal to the ratio of the number of workers hired in the formal sector to the vacancy rate.

$$q(\theta_t) = \frac{m(u_t, v_t)}{v_t} = k \left(\frac{1}{\theta_t}\right)^{\phi} \tag{8}$$

While the probability that an unemployed worker finds a job in the formal sector $\alpha(\theta_t)$ is defined by the ratio of matching workers in the formal sector to the unemployment rate.

$$\alpha(\theta_t) = \frac{m(u_t, v_t)}{u_t} = k \left(\theta_t\right)^{1-\phi} \tag{9}$$

The formal labor force evolves according to equation (3). In the period t there is a pool of unemployed workers that find a formal work with probability $\alpha(\theta)$, and there is a fraction of non-shirking workers who is fired with an exogenous separation rate $\mu \in (0,1)$. Also, the shirking workers has an additional exogenous probability of being unemployed defined by d

$$l_{t+1}^{f} = (1-\mu)l_t^{ns} + (1-\mu-d)l_t^{s} + \alpha(\theta)u_t \tag{10}$$

The production function of the formal firm depends on the worker. For simplicity, if the worker is non-shirking the production is given by $y_t^f = \psi^f$, where ψ^f is the formal firm productivity. But, the production is zero if the worker shirks. In addition, the formal firm has a fixed cost η to create a vacancy. Also, in each period, the formal firm pays profit tax defined by τ_t^{π} and pays a payroll tax τ^w to hire a worker. Then, the formal firms' net profits for each period t are given by:

$$\Pi_t^f = Div_t\,(1-\tau_t^{\pi}) = \left(y_t^f l_t^{ns} - (1+\tau_t^{w})w_t^f l_t^f - \eta v_t\right)(1-\tau_t^{\pi}) \tag{11}$$

Given the above, it is possible to get the three different states of the formal firms' profits along the time. The first possible state is represented by the value function of the formal firm to create a vacancy V_t is defined as follow:

$$V_t = -(1-\tau_t^{\pi})\eta + \left(\frac{1}{1+\rho}\right) E_t\left\{\Omega_{t+1}\left[q(\theta_{t+1})H_{t+1} + (1-q(\theta_{t+1}))V_{t+1}\right]\right\} \tag{12}$$

Where $H_{t+1} = J_{t+1}^{ns}$ if the worker chooses not to shirk, J_t^{ns} is the formal firm value function of filled job with a non-shirking worker. In contrast, $H_{t+1} = J_{t+1}^{s}$ in other case. That is, J_t^s is the formal firm value function of filled job with a shirking worker. The equation (10) represents the net cost of a vacancy will be $(1-\tau_t^{\pi})\eta$, also the value function have an expected probability $q(\theta_{t+1})$ in which the vacancy is filled by a worker, but with probability $1-q(\theta_{t+1})$ the vacancy is still open.

The value function of filling the vacancy with a non-shirking worker is J_t^{ns}. Hence, at the beginning of the period t the firm have a net profit given by $(1-\tau_t^{\pi})(y_t^f - w_t^f(1+\tau_t^{w}))$, and with an exogenous probability μ the worker is fired and the vacancy is open. In contrast, with $1-\mu$, the vacancy is still filled by a shirking or non-shirking worker.

$$J_t^{ns} = (1-\tau_t^{\pi})(y_t^f - w_t^f(1+\tau_t^{w})) + \left(\frac{1}{1+\rho}\right) E_t\left\{\Omega_{t+1}\left[\mu V_{t+1} + (1-\mu)H_{t+1}\right]\right\} \tag{13}$$

Differently, if the vacancy is filled with a shirking worker, the net profit of the firms does

not have production and is defined by $-(1-\tau_t^{\pi})(1+\tau_t^{w})w_t^f$. However, the probability of fired a worker and open a vacancy increase by d. In consequence, with $\mu + d$, the vacancy is open.

$$J_t^s = -(1-\tau_t^{\pi})(1+\tau_t^{w})w_t^f + \left(\frac{1}{1+\rho}\right) E_t \left\{\Omega_{t+1}\left[(\mu+d)V_{t+1} + (1-\mu-d)H_{t+1}\right]\right\} \quad (14)$$

In general terms, equations (11) and (12) show the different states of the formal firm profits along the time that depends on the worker state. If the worker is non-shirking, the production of the firm is given by y_t^f, and the probability of firing a worker is μ and open the vacancy. While if the worker is shirking, the firm's production is zero, but the probability of firing a worker is $\mu + d$ and, consequently, open the vacancy.

Informal Firms

Following Ulyssea (2018), the informal firm has a profit function defined in the equation (13). For simplicity I assume that the production of the informal firm is given by $y_t^i = \psi^i$. In this case, ψ^i is the productivity of the informal firm and by definition $\psi^f > \psi^i$.

$$\max \Pi^i = y_t^i l_t^i [1 - A(e_t)] - w_t^i l_t^i \quad (15)$$

Where l_t^i is the informal labor force in the period t with a law of motion given by:

$$l_{t+1}^i = (1-\sigma) l_t^i + \xi u_t \quad (16)$$

The probability that an unemployed was employed in the informal sector is exogenous and defined by ξ. While with probability σ, the informal worker is fired. Given Acosta(2020), the informal sector does not have any taxes, but is subject to an probability of auditing $A(e_t)$ that depends on enforcement expenditure by the government e_t.

$$A(e_t) = 1 - exp\left\{-\gamma e_t\right\} \quad (17)$$

I assume that $A(e_t)$ have a exponential distribution probability. Where $A_e(\cdot) > 0$. It means that an increase in the enforcement expenditure in the economy increases the probability of auditing an informal firm. Lastly, the informal wage is given by the marginal productivity in the informal firm and depends on the audited probability.

$$w_t^i = y_t^i (1 - A(e_t)) \quad (18)$$

2.3 Workers

Workers in the model are homogeneous. In the period t, a worker is in one of the three states, employed in the formal sector, employed in the informal sector, or being unemployed. However, workers in the formal sector could be employed as shirking workers or non-shirking workers. Given the above, the workers' utility has four possible states summarized in the following value functions.

$$W_t^{ns} = w_t^f - \chi - \zeta + \left(\frac{1}{1+\rho}\right) E_t \left\{\Omega_{t+1} \left[\mu U_{t+1} + (1-\mu) M_{t+1}\right]\right\} \tag{19}$$

The equation (17) shows the utility along the time if the worker is employed in the formal sector and is non-shirking, where $M_{t+1} = \text{Max}\{W_{t+1}^{ns}, W_{t+1}^{s}\}$. In this case, at the beginning of the period t the worker has a real wage given by w_t^f but suffer a disutility in term of the real wage given by ζ to decide to be non-shirking, and there is another disutility to work χ. Also, with probability μ, the worker is fired from the formal sector and becomes unemployed.

In contrast, if the worker is shirking does not has a disutility ζ, but have the disutility to work given by χ, so utility at the beginning of the period t is equal to $w_t^f - \chi$. Nevertheless, the probability of being fired and become unemployed increases and is equal to $\mu + d$. The value function of being employed in the formal sector as a shirking worker is given by:

$$W_t^{s} = w_t^f - \chi + \left(\frac{1}{1+\rho}\right) E_t \left\{\Omega_{t+1} \left[(\mu+d) U_{t+1} + (1-\mu-d) M_{t+1}\right]\right\} \tag{20}$$

The value function of a worker who is employed in the informal sector is defined in equation (19). At the beginning of the period t, the worker has a utility of $w_t^i - \chi$ and has an exogenous probability of being fired from the informal work given by σ.

$$W_t^{i} = w_t^i - \chi + \left(\frac{1}{1+\rho}\right) E_t \left\{\Omega_{t+1} \left[\sigma U_{t+1} + (1-\sigma) W_{t+1}^{i}\right]\right\} \tag{21}$$

Similarly, the value function of being unemployed is defined by:

$$U_t = s_t + \left(\frac{1}{1+\rho}\right) E_t \left\{\Omega_{t+1} \left[\alpha(\theta_{t+1}) M_{t+1} + \xi W_{t+1}^{i} + (1 - \alpha(\theta_{t+1}) - \xi) U_{t+1}\right]\right\} \tag{22}$$

In this case, if the worker is unemployed, have a government transfer of s_t, and the endogenous probability of being employed in the formal sector is $\alpha(\theta_{t+1})$ while the

probability of being employed in the informal sector is exogenous and equal to ξ. On the other hand, the worker continue be unemployed with probability $1 - \alpha(\theta_{t+1}) - \xi$.

2.4 Government

In each period, the government is represented by a balanced budget rule as follow:

$$g_t + e_t + s_t u_t = \tau_t^w w_t^f l_t^f + y_t^i l_t^i A(e_t) + \tau_t^\pi Div_t \tag{23}$$

The government expenditures is given by the unproductive expenditure g_t, the enforcement expenditure e_t, and the unemployment transfers $s_t u_t$. The government income is defined by payroll taxes from the formal sector $\tau_t^w w_t^f l_t^f$, profit taxes from formal firms $\tau_t^\pi Div_t$, and income from the informal sector if informal firm is audited $y_t^i A(e_t)$.

In baseline case of the model, I assume that the unproductive expenditure g_t is endogenous, and the tax policies τ_t^w and e_t are exogenous. The above implies that any change in one of the tax policies remains the other constant lead to a change in the unproductive expenditure.

$$\begin{aligned} \Delta\tau_t^w \quad &\rightarrow \quad \Delta g_t,\ \overline{e}_t \\ \Delta e_t \quad &\rightarrow \quad \Delta g_t,\ \overline{\tau}_t^w \end{aligned}$$

Nevertheless, in the following simulations of the model, I consider the case when the enforcement expenditure e_t and the payroll taxes τ_t^w are endogenous. Hence, any change in one of the tax policies remains the unproductive expenditure constant, lead to a change in the endogenous tax policy.

$$\Delta\tau_t^w \quad \rightarrow \quad \Delta e,\ \overline{g}$$

2.5 Steady State

Given the above, the model's analysis focuses on the steady-state to understand the role of the real wage rigidities given the tax policies that encourage the formality in the long run. For this purpose, I compare different scenarios in the long run for the interest variables under changes in the tax policies.

Firms

In the steady-state, the value function of the formal firms described in the equations (10) - (22) is presented as follow:

$$\rho V = -(1-\tau^{\pi})\eta(1+\rho) + q(\theta)\left(J^{ns} - V\right) \tag{24}$$

Given the steady-state in the economy, in the value function of creating a vacancy, I assume that with probability $q(\theta)$ the vacancy is filled with a non-shirking worker, and the cost of creating a vacancy is affected by a profit tax.

$$\rho J^{ns} = (1-\tau^{\pi})(y^{f} - w^{f}(1+\tau^{w}))(1+\rho) + \mu\left(V - J^{ns}\right) \tag{25}$$

In the same way, the formal firm value function of filling a vacancy with non-shirking and shirking workers is defined in the equation (23) and (24).

$$\rho J^{s} = -(1-\tau^{\pi})(1+\tau^{w})w^{f}(1+\rho) + (\mu+d)\left(V - J^{s}\right) \tag{26}$$

From the side of the informal firm the informal wage in the steady-state is given by:

$$w^{i} = y^{i}(1 - A(e)) \tag{27}$$

Workers

The value functions of the workers summarized in the equations (17) - (20) in the steady-state are the following:

$$\rho W^{ns} = (w^{f} - \chi - \zeta)(1+\rho) + \mu(U - W^{ns}) \tag{28}$$

If the worker is formal, then the value functions could be of the non-shirking or shirking worker. In this sense, I assume that with probability μ, the non-shirking worker becomes unemployed, and with probability $\mu + d$, the shirking worker becomes unemployed.

$$\rho W^{s} = (w^{f} - \chi)(1+\rho) + (\mu+d)(U - W^{s}) \tag{29}$$

When the worker is informal, the value functions in steady-state is the following:

$$\rho W^{i} = (w^{i} - \chi)(1+\rho) + \sigma(U - W^{i}) \tag{30}$$

Finally, when the worker is unemployed, the value functions are presented below. In this case, I assume that in a steady-state the unemployed become a formal non-shirking worker with probability $\alpha(\theta)$

$$\rho U = s(1+\rho) + \alpha(\theta)(W^{ns} - U) + \xi(W^{i} - U) \tag{31}$$

It is essential to highlight that there is no job-to-job transition in the setup of the model. To change the state, the worker must first be unemployed and become a formal or informal worker. The worker does not have a direct transition from formal to informal or informal to formal state.

Real Wage Rigidities

The shirking mechanism determines the formal wage in the model. That is, non-shirking workers suffer a disutility ζ when is employed. If this disutility increase, the workers will have an incentive to shirk the work. However, formal firms do not want to hire shirking workers. Given the above, the formal firm has the incentive to increase the formal wage w^f, and this increase is above the equilibrium wage with any distortion in the market. Hence, the formal wage has real wage rigidities that are persistent along the time.

To determine the real wage and follow the standard literature of the shirking models, I assume the free entry condition where the value of creating a vacancy is equal to zero $V = 0$. From equation (22) is possible to get:

$$J^{ns} = \frac{(1-\tau^{\pi})\eta(1+\rho)}{q(\theta)} \tag{32}$$

Replacing the equation (30) in equation (23) is possible to obtain the formal wage offer by the firm:

$$w^f = \left(y^f - (\mu+\rho)\left(\frac{\eta}{q(\theta)}\right)\right)\frac{1}{1+\tau^{w}} \tag{33}$$

Using the non-shirking condition $W^{ns} = W^{s}$ and using the equations (26), (27), and (29) is possible to obtain the wage from the workers' side.

$$w^f = \left(\rho + \mu + \frac{(\rho+\sigma)\alpha(\theta)}{\rho+\sigma+\xi}\right)\frac{\zeta}{d} + \Gamma \tag{34}$$

$$\Gamma = \chi + \zeta + s + \frac{\xi}{\rho+\sigma+\xi}\left(y^{i}(1-A(e)) - \chi - s\right)$$

Equation (32) shows that the informal wage affected the formal wage and depends on the probability that an unemployed is employed in the informal sector ξ. Also, the rigidities in the formal wage are given by the disutility of being a non-shirking worker ζ. The formal wage in the model with informality is a generalized version of the Shapiro Stiglitz wage, consider the case in which there is not informal sector, and lets assume that the probability of being an informal worker is equal to zero $\xi = 0$ the formal wage become:

$$w^f = (\rho + \mu + \alpha(\theta)) \frac{\zeta}{d} + \chi + \zeta + s \tag{35}$$

Beveridge Curve

The classic search and matching model makes it possible to obtain the Beveridge curve that shows the relationship between the vacancy rate and the unemployment rate. For the above, consider the equations of law motion of labor force in the formal and informal sector (8), (14), and the equation of the total labor force in the steady-state.

$$\begin{aligned} 1 &= l^f + l^i + u && (36)\\ \mu l^f &= \alpha(\theta) u && (37)\\ \sigma l^i &= \xi u && (38) \end{aligned}$$

Whit the above equations is possible to get the classic Beveridge curve defined as follow:

$$u = \frac{\mu}{\alpha(\theta) + \frac{\xi+\sigma}{\sigma}\mu} \tag{39}$$

The equation (37) is a generalized version of the Beveridge curve. Let us assume that there is not informal sector $\xi = 0$ is possible to return the Beveridge curve of the simple search and matching model.

$$u = \frac{\mu}{\alpha(\theta) + \mu} \tag{40}$$

Additionally, the informal sector and the equations (34) - (36) are possible to get a new version of the Beveridge curve that related the formal vacancy with the informality and the formal vacancy with the non-formal sector. The above is a replication of the stylized fact that it is possible to see a non-linear relationship between formal vacancy and informal rate.

$$u + l^i = \frac{\mu}{\mu + \left(1 - \left(\frac{\xi}{\sigma+\xi}\right)\right)\alpha(\vartheta)} \tag{41}$$

Where $\alpha(\vartheta) = k\left(\frac{v}{u+l^i}\left(\frac{\sigma+\xi}{\sigma}\right)\right)^{1-\phi}$ with $\vartheta = \frac{v}{u+l^i}$. To get the equilibrium from Beveridge curve side I use the equation (41) and the following equation.

$$v = \theta\frac{\sigma}{\sigma+\xi}(u + l^i) \tag{42}$$

Hence to understand who the formality could be change given different tax policies, in the following sections, I use the equation (31), (32), (40), and (41) to get the equilibrium.

Equilibrium Condition

Given the workers and firms optimization summarized in the steady states value functions, the labor market equilibrium of the economy is characterized by the equations system equations (31), (32), (40), and (41). With the above, it is possible to get the real formal wage of equilibrium w^{f*} and the labor market tightness of equilibrium θ^*.

Hence, it is possible to obtain the formal and informal workers in equilibrium and the unemployed workers. Also, given that the economy does not have capital, the bonds in equilibrium are equal to zero $b = 0$. Replacing the informal firm's optimal condition from the equation (25) and the steady-state government balance on the steady-state budget balance of household is possible to find the equilibrium balance defined as follow:

$$GDP = y^f l^f + y^i l^i - \eta v = c + g + e \tag{43}$$

Figure (1) represents the equilibrium in the economy in which the intersection of curves (32) and (31) returns the formal wage of equilibrium w^* and the labor market tightness of equilibrium θ^*. Also, with the equations (40), (41), and the labor market tightness of equilibrium, it is possible to obtain the formal vacancy rate of equilibrium v^* and the unemployment plus the informal rate of equilibrium $(u + l^i)^*$

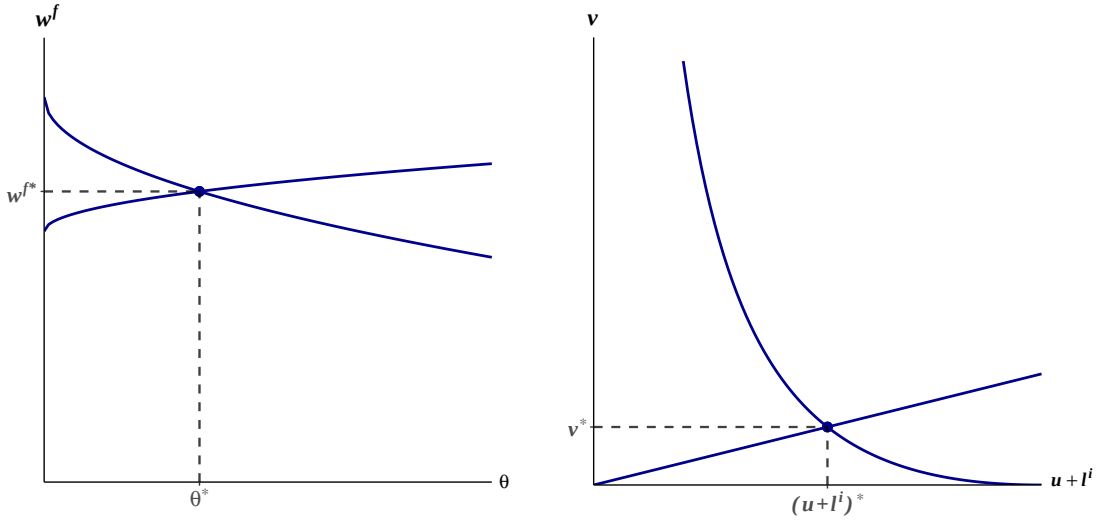

Figure 1: Equilibrium in the economy

3 Calibration

The calibration of the model seeks to adjust to the main characteristics of the Colombian labor market, and the period is set to one month. For this reason, I divide the parameters into two groups. In the first group are parameters that are difficult to identify with the data. Consequently, I use typical values from search and matching models that incorporate an informal sector in the Colombian economy or parameters from the international literature.

Given the above, following Granda Carvajal & Hamann (2020), I assign the value of the discount rate ρ equal to $\left(\frac{1}{0.972^{1/12}} - 1\right)$. For the parameters describing matching and frictions, I use the standard values in the literature, $\phi = 0.5$, $\eta = 0.5$, and $k = 0.25$ used in Albrecht et al. (2009). Regarding the parameters of labor market dynamics are set as follow: for simplicity, I assume the unemployment transfer $s = 0$; the formal and informal separation rate are $\mu = 0.0244$ and $\sigma = 0.0235$ given estimations of M. Gómez et al. (2021). Also, due to the difficulty of estimating the disutility of working, I use the Martin & Wang (2020) value $\chi = 0.62$.

As for the tax fiscal policy parameters benchmark, I set the value of τ^w and τ^i as 0.33 and 0.13, respectively, following Rincón-Castro (2021). The enforcement expenditure and the probability of being audited are taken from Posada & Mejía (2012), who develop a model with informal sector and enforcement policies, hence $e = 0.007$ and $A(e) = 0.27$

Parameter	Description	Source	Value
ρ	Discount rate	Granda Carvajal & Hamann (2020)	0.009
k	Matching process efficiency	Albrecht et al. (2009)	0.25
ϕ	Matching elasticity	Albrecht et al. (2009)	0.5
s	Unemployment transfer	Albrecht et al. (2009)	0
η	Vacancy cost	Albrecht et al. (2009)	0.5
μ	Formal separation rate	Morales et al. (2019)	0.0244
σ	Informal separation rate	Morales et al. (2019)	0.0235
τ^w	Payroll tax	Rincón-Castro (2021)	0.33
τ^π	Income tax	Rincón-Castro (2021)	0.13
e	Enforcement expenditure	Posada & Mejía (2012)	0.007
$A(e)$	Auditing probability	Posada & Mejía (2012)	0.27
χ	Disutility of working	Martin & Wang (2020)	0.62
d	Detection rate	Calibrated	0.96
ζ	Disutility of effort	Calibrated	0.29
ξ	Prob of working informally	Calibrated	0.08
γ	Institutional efficiency	Calibrated	42.44
θ	Labor market tightness	Calibrated	0.17
y^i	Informal production	Calibrated	1
y^f	Formal production	Calibrated	1.37

Table 1

The second set of parameters is calibrated to match the average unemployment, formal and informal labor rates observed in the data from 2008 to 2019 and normalizing the informal production $y^i = 1$. Using the equilibrium equations described in the previous section, I select the value of ζ and d to match the unemployment rate $u = 0.112$, the formal rate $l^f = 0.478$, and the informal rate $l^f = 0.410$[1]; the ratio between formal and informal wage observed in the data $\frac{w^f}{w^i} = 1.39$[2]

Finally, the probability of working in the informal sector ξ and labor market tightness θ is derived from the labor market equations (34) – (36). The institutional efficiency γ is obtained using the compliance probability equation (15), and the formal production is derived using the equation (33). The result of the calibrated parameters is presented in the Table 1.

[1]The average rates are calculated based on information from the Colombian Household Surveys (GEIH, for its acronym in Spanish) published by the National Statistics Department (DANE, for its acronym in Spanish) for the 23 main metropolitan areas.

[2]The relationship between formal and informal wages was estimated using Mincer equations using data from the GEIH from 2017 to 2019

4 Policies

This section explores the long-run tax policies effect defined as the change of payroll taxes and the change in enforcement expenditure on the main variables in the labor market, taking to account the role of the real wage rigidities on the effect of tax policies. In the first part, I analyze the change in equilibrium variables assuming an endogenous government expenditure, which adjusts for payroll taxes or enforcement expenditure changes.

In the second part, I simulated the model to get the long-run effects of change in payroll taxes and enforcement expenditure assuming an endogenous public expenditure. On the other way around, I simulated the model when the public expenditure is constant and the enforcement expenditure is endogenous

4.1 Analytical Results

Using the model's equilibrium, I assume a decrease in the payroll taxes with high and low real wage rigidities. Figures (2) and (3) show the effect in the equilibrium with high and low real wage rigidities, respectively. When the economy is in front of high wage rigidities, the policy's effect is lower than the scenario in which it has low wage rigidities.

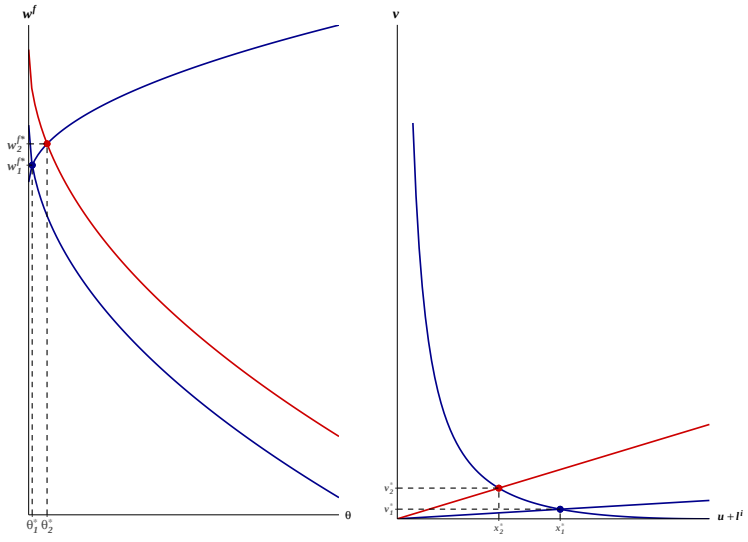

Figure 2: Decrease payroll tax with high wage rigidities

In both scenarios, the decrease in the payroll taxes increases the formal wage w^f and the labor market tightness θ. The increase in θ, lead to an increase in the vacancy rate and a decrease in the unemployment and informal rate. Nevertheless, in figure (2), the decrease in unemployment and informality is higher than the figure (3).

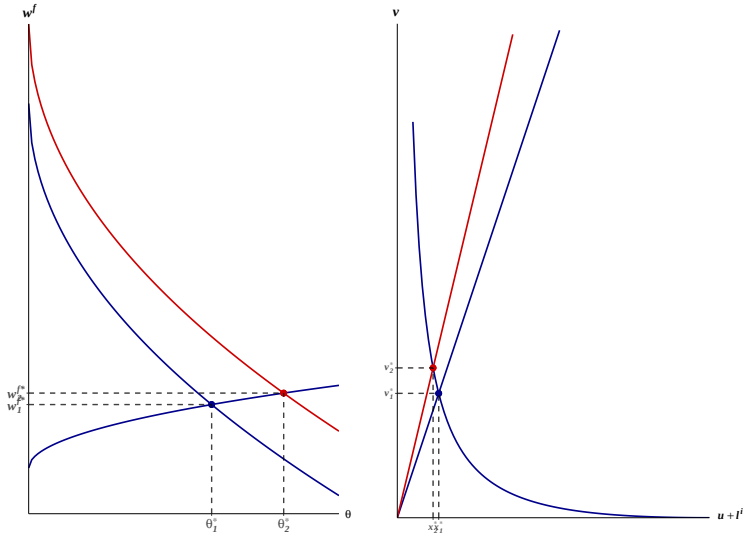

Figure 3: Decrease payroll with low wage rigidities

On the other hand, the figures (4) and (5) show the effect on the economy when the government increase the enforcement expenditure. In both cases, the increase in the enforcement expenditure lead to a decrease of the formal wage w^f with an increase in the labor market tightness θ. The above, induce an increase in the vacancy rate, and a decrease in the unemployment and the informality.

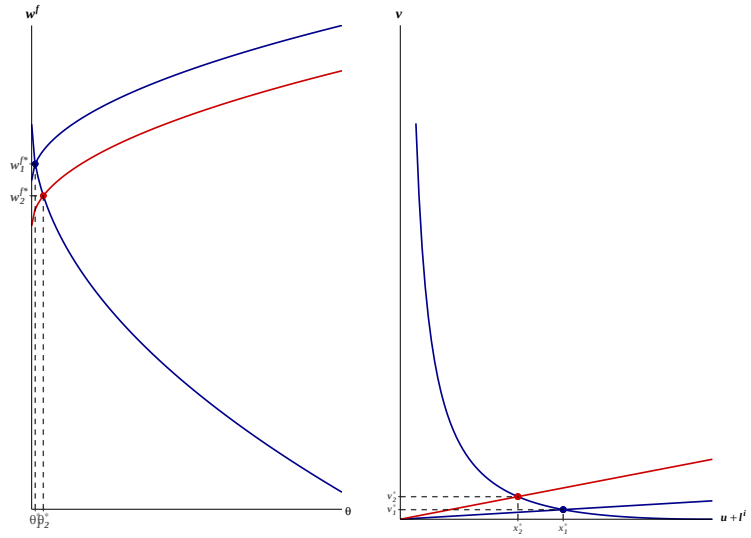

Figure 4: Increase enforcement expenditure with high wage rigidities

The analytical results suggest that an increase in enforcement expenditure has a more significant effect in reducing informality and unemployment when the economy faces higher wage rigidities. For both tax policies (decrease payroll taxes and increase enforcement expenditure), when the economy has lower wage rigidities, the relative reduction of informality and unemployment is lower but is higher in the absolute term. In contrast, when the economy has high wage rigidities, the relative reduction of informality and unemployment is higher but lower in absolute terms.

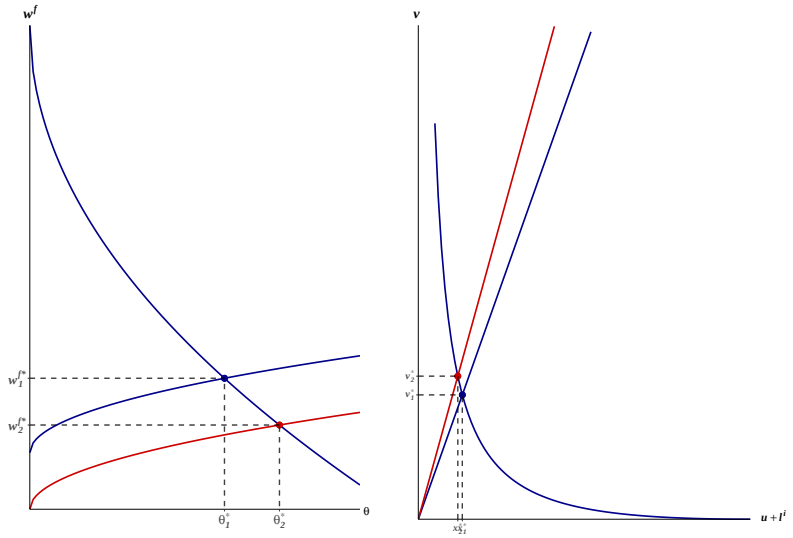

Figure 5: Increase enforcement expenditure with low wage rigidities

4.2 Simulations

The simulation of the model presents the primary outcome variables in the long run, given the change of the tax policies. Figures (6) and (7) estimate the model with endogenous public expending. Hence, the government expenditure adjusts for changes in the payroll tax and enforcement expenditure. In contrast, Figure (8) shows the simulation results when the public expending in the economy is constant, and the enforcement expenditure is endogenous. In both simulations is presented the case when the economy is in front of high real wage rigidities and low real wage rigidities.

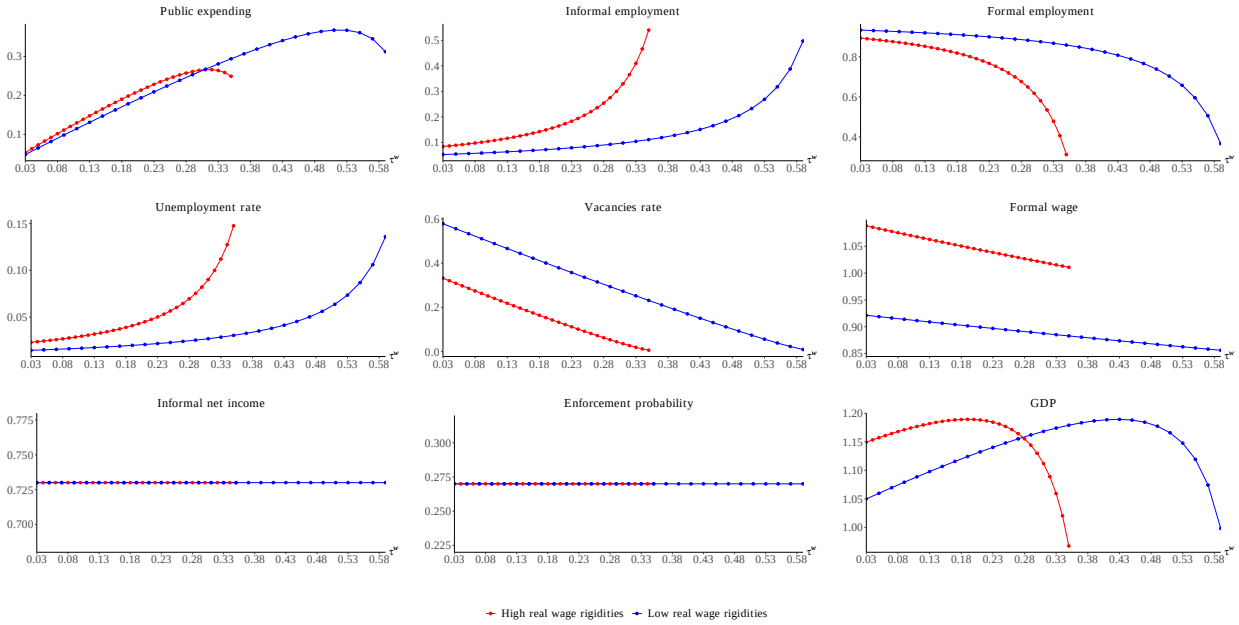

Figure 6: Multiple payroll tax policies with constant enforcement expenditure, each point is a steady-state value of the variable given the value of τ^w.

Figure (6) show the simulations results for multiple payroll tax policies. The increase in the payroll taxes lead to an increase in the public expending, with an increase in the informal employment and unemployment. The informal employment increase more slowly when the economy have low real wage rigidities with an increase in the payroll taxes. The laffer curve of public expending suggest that exist a policy in which is possible decrease the payroll taxes, increase the public expending (proxy of government income) and reduce the informality.

This reduction in the payroll taxes, has a higher public expending when the economy in more flexible. However, when the economy have more rigidities in the labor market the effect of reduce the payroll taxes has a higher effect in reduce the informality in relative terms. Also, this reduction could be increase the GDP of the economy with high and low real wage rigidities.

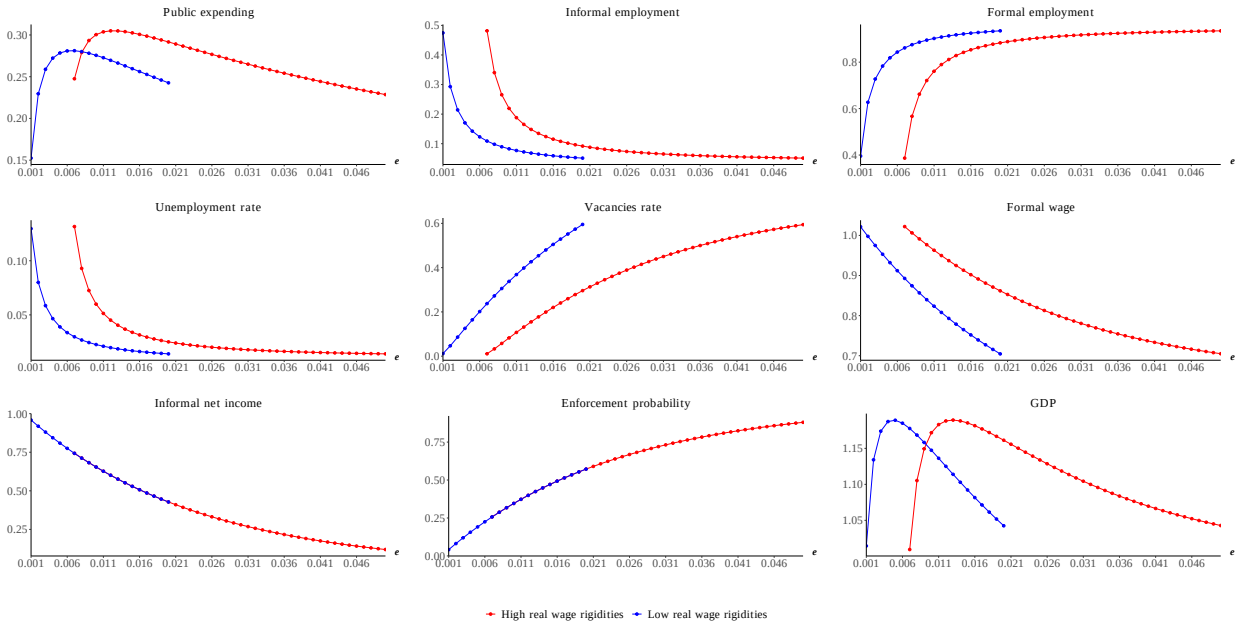

Figure 7: Multiple enforcement expenditure policies with constant enforcement expenditure, each point is a steady-state value of the variable given the value of e.

Concerning the enforcement expenditure policy, figure (7) shows the results of the simulation. When the economy has high real wage rigidities, the public expenditure is higher than those with low wage rigidities. Also, the increase in enforcement expenditure leads to a decrease in informality and unemployment. The reduction of informal labor is higher when the economy has low wage rigidities.

The GDP increase with the increase in enforcement expenditure e; however, there is a pike in which an additional increase in e induces a decrease in the GDP. When the economy has low wage rigidities, the increase in enforcement expenditure generates an increase in GDP, reaching it is maximum faster.

Finally, figure (8) shows the simulation results of a constant public expending when the enforcement expenditure is adjusted for changes in the payroll taxes. These simulations allow understanding the combination of tax policies effect on the labor market. The enforcement expenditure has an inverted ”U” shape. Hence the increase in the payroll

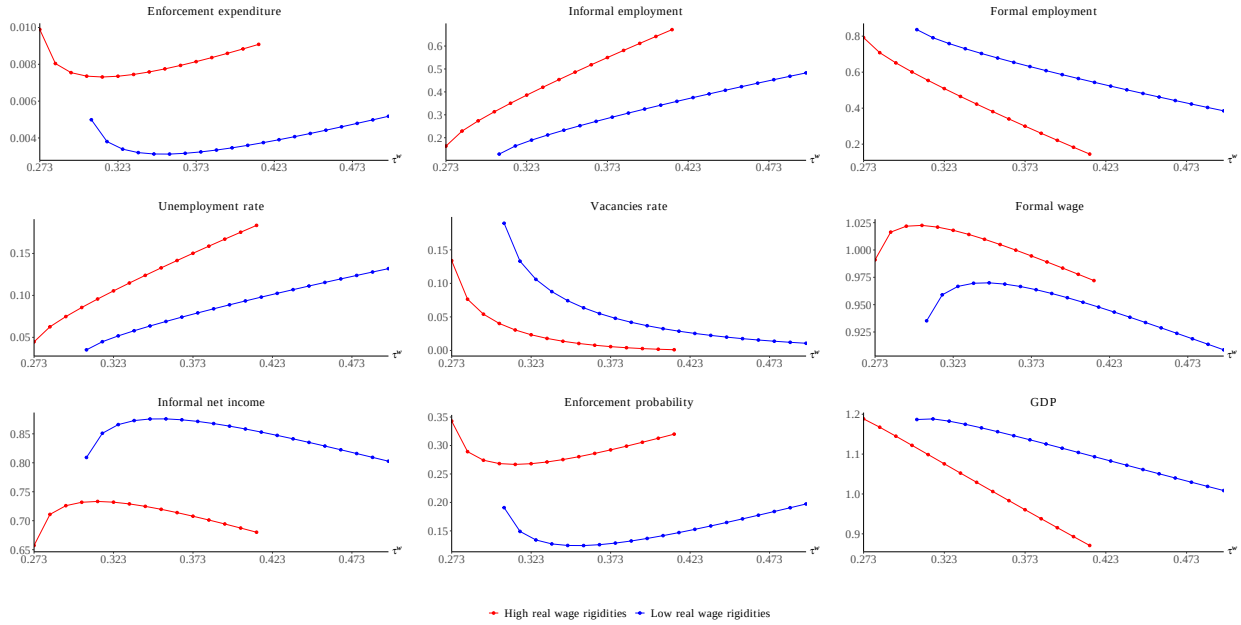

Figure 8: Multiple payroll tax policies with constant public expending, each point is a steady-state value of the variable given the value of e.

taxes leads to a decrease in the enforcement expenditure. However, there is a point in which the increase in payroll taxes generates an increase in enforcement expenditure.

The results show that the increase in the payroll taxes induces an increase in informality, no matter the enforcement expenditure. The simulation could suggest a combination of tax policies, in which it is possible to reduce the payroll taxes and increase the enforcement expenditure to reduce the informality significantly. However, the increase in enforcement expenditure is higher when the economy has high real wage rigidities. Also, the GDP decrease with the increase of payroll taxes and endogenous enforcement expenditure.

5 Conclusion

This article has developed a dynamic general equilibrium model with search and matching frictions and rigidities of real wages through circumvention mechanisms, based on Martin & Wang (2020). It includes a government and an informal labor market. The simulation results suggest that the rigidities of the real wage in the economy are a relevant determinant of the magnitude of the tax policies that seek to reduce informality.

In relative terms, when the economy has high rigidities in real wages, a decrease in

payroll taxes has a more significant effect in reducing informality. While, when there are low real wage rigidities in the economy, the increase in the enforcement expenditure leads to a greater reduction in informality.

Also, the model shows that for each tax policy in the paper, there is a scenario in which it is possible to reduce the informality with an increase in the government expenditure (proxy of government income). When there is a reduction of payroll taxes, the increase in public expending is higher for the economy with low real wage rigidities. Nevertheless, the increase in enforcement expenditure leads to a higher public expending when the economy is in front of higher real wage rigidities.

When the public spending is fixed, the simulation results suggest a combination of tax policies (increased enforcement expenditure and reduced payroll taxes) that could reduce the informality in an effective way. The combination of policies depends on the real wage rigidities in the economy. With higher real wage rigidities is necessary more enforcement expenditure.

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