Temporary employment, informality, poverty and inequality

José Manuel Tapias Ortega

Documento de Trabajo
Alianza EFI - Colombia Científica
Diciembre 2021

Número de serie: WP5-2021-010

Temporary employment, informality, poverty and inequality

José Manuel Tapias Ortega
Universidad del Rosario

December 3, 2021

Abstract

An essential tool in the fight against poverty is the generation of decent jobs through an equitable and inclusive labor market. To achieve this, two fronts of struggle must be addressed. The first is informality, which causes many workers in developing countries to have precarious working conditions and low incomes. The second refers to the misuse of temporary contracts. Temporary jobs are associated with instability and wage penalties. Therefore, a policy aimed at obtaining decent jobs to overcome poverty must include these two battlefronts. In this sense, a Computable General Equilibrium (CGE) model with temporary, permanent, and informal work is developing. We show some interesting relationships through the equations formulated so far. The future results of this work will be helpful to address and generate policies aimed at achieving decent jobs.

Keywords Labor informality, temporary employment, poverty, inequality, macroeconomics.

JEL classification E24, E26, I3, D63.

Acknowledgment Thanks Alianza EFI-Colombia Científica grant with code 60185 and FP44842-220-2018. Also, thanks to Fernando Jaramillo who is the thesis director of this work, whose valuable comments and guidance have been essential for the development of this work.

1 Introduction

The fight against poverty is one of the main issues on the agenda of developing countries. In this sense, one of the principal means to win this fight is creating an equitable and inclusive labor market. One of the main obstacles to having a labor market with such conditions is the persistence of informality, which ranges between 90% and 30% in developing countries (Bacchetta, Ernst, Bustamante

et al. 2009). These figures are quite high compared to the levels of informality in more developed countries, where informality is around 18% (Bonnet, Vanek & Chen 2019). Informality prevents informal workers from accessing social security and mechanisms to enforce their rights. Similarly, informal enterprises have low productivity and no access to financing channels (ILO 2018*b*).

Accordingly, the generation of formal jobs helps expand access to social protection and represents a stable source of income that allows workers to save and invest in assets. However, there are currently some forms of formal employment that do not meet all of these conditions, especially in terms of stability, which means that they do not improve workers' living conditions to the extent that they should. Some of these forms of formal employment are included in what are known as non-standard forms of employment, which are particular types of arrangements that differ from standard employment (ILO 2016). This term includes employment modalities such as temporary employment, part-time or disguised employment, self-employment, multiparty employment relationships, and economically dependent self-employment. Moreover, like informality, these forms of employment negatively affect the labor market because they are related to increased unemployment, wage penalties, uncertain wages, low probability of receiving training, and low productivity. Hence, the widespread use of non-standard forms of employment, which are related to low-quality working conditions, becomes another obstacle to an inclusive labor market that contributes to reducing poverty and inequality.

Thus, the coexistence of these two obstacles makes their respective adverse effects reinforce each other, making it more difficult for workers to escape poverty. For this reason, the policies aimed at succeeding at any of these challenges must be done considering these interrelationships (ILO 2018*a*). A formal labor market with high levels of non-standard forms of employment is not attractive to informal workers, which makes formalization more difficult. Furthermore, productivity losses due to hiring under non-standard forms can cause a firm to become informal. In this vein, informality and the improvement of working conditions in non-standard forms of employment are one of the key challenges of developing countries in order to achieve decent jobs. However, the relationship between these two phenomena has been little studied directly and the consequences of policies aimed at improving the effects of non-standard work on informality and vice-versa are not very clear in terms of labor market results, poverty and inequality.

Although it is interesting to study these two phenomena, non-standard jobs encompass classes of jobs that differ considerably from each other. Consequently, this paper will focus on the study of informality and temporary jobs. Temporary work will be understood as fixed-term, project- or task-based formal contracts, which are contractual agreements between an employer and an employee characterized by a limited duration or a predefined event that terminates the contract. Such contracts may be of a commercial or labor nature.

Temporary work is interesting for three reasons. Firstly, it is a method of hiring that represents advantages for companies since it allows them to have cheaper workers, who can be fired without cost at the end of the contract. Secondly, informal workers who observe that the probability of being a temporary worker is high in the formal market may decide not to join the formality because of the few advantages of this form of hiring. Finally, this type of employment is observable through surveys, which facilitates its study.

Likewise, temporary employment is not necessarily a problem if used appropriately, since companies require flexibility in hiring due to the growth of the service sector and to globalization, and workers need to reconcile work life with the opportunity to access education, raise children, and care for the elderly. Nevertheless, temporary workers are likely to experience job instability, fewer opportunities for promotion, few training opportunities, and lower wages (Maurizio 2016). In fact, in Latin America the penalty associated with temporary employment can be as high as 15% (Maurizio 2019). In addition, this is combined with a percentage of informal employment amounting to 53,1% in the region (OECD & ILO 2019).

In this regard, Colombia is one of the most interesting countries to study this phenomenon due to its high temporary employment and informality levels. In 2013, 73% of the Colombian population between 15 and 64 years of age was active in the labor market and 66,5% was employed. Those were favorable figures compared to OECD member countries, whose averages were 71% and 65,3% respectively (OECD 2016). However, those results contrast with a worrying urban informality rate of 48,3% (third quarter of 2017) (FEDESARROLLO 2018). In addition, it is estimated that in 2014 the percentage of temporary employment in the Colombian industry was of 40,6% (Rodríguez & López 2016).

In Colombia, temporary contracts guarantee social security to employees, but provide fewer benefits compared to open-ended contracts, which means that it is cheaper to hire a temporary worker. As a result, employers have incentives to use these contracts to reduce costs and not to use them to adapt to the cyclical changes (ILO 2016). This makes the formal labor market unattractive to workers since, upon entering it, they are very likely to be hired as temporary workers, so they might prefer to be informal. Therefore, the goal is to generate the right incentives and ensure that companies have access to these contracts so that they can adapt to the economic cycle, but without abusing their use, making it easier to implement policies aimed at reducing informality. It is necessary to ensure that such flexibility is not at the expense of workers and that, as far as possible, workers' conditions do not depend on the type of contract to generate decent jobs. One way to contribute to this is to increase the benefits of temporary workers by making the cost of firing them similar to that of firing a permanent worker. Thus, focusing on the Colombian case, this research will attempt to answer the following question: What is the effect of improvements in working conditions of temporary employment on the labor market results,

informality and poverty? This study is important for Colombia and other countries with similar levels of development in which the negative effects of high levels of informality coexist with the effects of temporary employment. We will use a Computable General Equilibrium Model (CGE) with microsimulations to analyze the problem.

This document is divided into 7 parts in addition to this introduction. In the second, there is a brief review of the literature, which is incomplete and should not be given much attention. In the third part, the figures on informality and temporary work in Colombia are studied with the aim of verifying that the figures are a worrying phenomenon, and to raise the first intuitions that will serve as input for the theoretical model. In the fourth part, the methodology will be described in a general way. In the fifth part, the CGE model will be described with emphasis on the labor market. In the sixth part, what is planned to be done in the microsimulation part will be briefly described. In section 7, preliminary results of a simulation exercise with fictitious data are presented. Finally, some preliminary conclusions about Colombia and the model are raised in the last section.

2 Literature review

The effects of variations in firing costs of temporary workers on the labor market and poverty in the presence of informality have not been studied to our knowledge. However, some studies have studied some of these relationships separately. Hence, the information will be organized as follows. Firstly, papers that address the relationship between temporary jobs and labor market outcomes will be mentioned. Finally, some papers dealing with the effects of different economic policies on informality, poverty or the labor market are described.

The relationships between temporary jobs and the labor market have been studied both from an empirical perspective and from dynamic models and search and matching models. For example, Blanchard and Landier (2002) studied the effect of a reform that made it possible to hire workers on fixed-term contracts in the French labor market. Using a search and matching model, the authors found that this measure led to increased turnover and unemployment. They also show that even if unemployment does fall workers may have it worse, going through several periods of unemployment and entry level jobs before getting a regular job. In addition, Cahuc and Malherbet (2016) found that the protection of permanent jobs does not significantly affect employment, but causes the substitution of permanent jobs by temporary ones, leading to a decrease in the economy's total output decrease and in productivity.

Similarly, Booth and Marco (2002) find, using panel data analysis, that in England, temporary workers have lower satisfaction levels, receive less training,

and are paid less well than permanent workers. They also find that temporary workers are a stepping stone to permanent positions in this country. Equally, Boeri and Pietro (2007) showed that a labor reform that allows coexistence between temporary and permanent contracts increases job creation on a transitional basis, but labor productivity decreases in this transition. The authors used a dynamic model of labor demand under uncertainty.

On the other hand, Cahuc and Postel-Vinay (2002) show that the combination of strong labor protection of permanent jobs and the introduction of temporary jobs in Europe increases unemployment and reduces welfare because these measures have opposite effects on job destruction and job creation. Moreover, Cappellari, Dell'Aringa and Leonardi (2012) perform empirical analysis to study the effects of two reforms on different types of temporary contracts in Italy. First, they find that stimulating the hiring of apprentices by relaxing training requirements and extending their use to people up to 30 years old increases hiring through these contracts and increases productivity. Finally, they show that making firms not have to write the specific reason for using temporary workers in the contract leads to higher turnover and lower productivity

Furthermore, Castellani, Lotti and Obando (2020) empirically analyze the effect of hiring modalities on firm productivity in the industrial sector in Colombia. They find that the income elasticity for temporary workers compared to that of permanent workers is lower. However, some forms of flexibility may induce an increase in productivity, but only in small firms. Dolado, Garcia and Jimeno (2002) found, empirically, that in Spain, allowing the use of temporary workers to do permanent activities and lowering the cost of firing led to an increase in turnover rate and an increase in employment. However, investment in human capital declined, and wage dispersion increased.

Also, Maurizio (2019) analyzed the effect of temporary employment on wages in Latin America. He showed that temporary jobs are associated with low stability, precarious working conditions, and low wages. In fact, it shows that the penalty associated with being a temporary worker in the region can be as high as 15%, which increases inequality. Moreover, Eslava, Haltiwanger, and Kugler (2014) found similar results for Colombia. In this country, temporary contracts are associated with greater business volatility, less need for skilled labor, and lower productivity in firms that are intensive in skilled labor. In addition, Engellandta and Riphahn (2005) empirically demonstrated that temporary workers have greater incentives to work hard, whereas permanent workers have less incentive to work hard due to labor protection.

Instead, Hopenhayn and Rogerson (1993) build a dynamic model in order to analyze the relationship between a job destruction tax and employment, but without distinguishing between permanent and temporary contracts. The authors conclude that the job destruction tax decreases total employment because it lowers productivity. Furthermore, Kleinknecht et al. (2009) studied the Dutch

job creation miracle during the 1980s and 1990s. The authors argue that even though temporary contracts lead to wage cost savings causing employers to hire more employees, this does not necessarily translate into increased sales, which decreases productivity. Lastly, Kluger (2007) analyzes the effect of job security regulations on the labor market through a difference-in-differences estimation and a search and matching model in the presence of informality. Using informal workers as the control group and formal workers as the treated group, he finds that reducing firing costs increases turnover and decreases unemployment.

On the other hand, there are a considerable number studys that have analyzed the effect of different policies on informality, poverty, and the labor market. For instance, It has been demonstrated that informality in the intensive margin (when formal firms hire informal employees) and informality in the extensive margin (whether the company is registered or not) do not always go in the same direction in the face of policies aimed at formalizing businesses. Also, The same study found that a decline in informality is not necessarily associated with an improvement in well-being (Ulyssea 2018). In addition, other research has found that, although increased enforcement does not increase unemployment, it increases wages, total output and welfare by allowing a better allocation of workers to higher productivity jobs and improving competition in the formal labor market (Meghir, Narita & Robin 2015).

Moreover, El Badaoui and Magnani (2019) study the relationship between informality, poverty and inequality through a micro-macro simulation approach in South Africa. They showed that a sufficiently high flat tax rate combined with a transfer paid only to workers reduces the informal sector, inequality and poverty. Equally, Agénor et al. (2006) develops a CGE model for Brazil indicating that the increase in interest rates increases unemployment, poverty and inequality. It also causes the wage differential between the informal and formal sectors to decrease, resulting in an increase in the labor supply in this sector. Additionally, Bussolo and Lay (2006) explain that trade liberalization in Colombia reduced poverty and informality. The authors used a CGE model with microsimulations.

Conversely, Akbulut and Taylan (2020) argue that reducing import taxes increases informality and unemployment in Turkey, whereas increasing import tariff rates expands formal employment. Bourguignon and Savard (2008) create a microsimulated CGE model for the Philippines. They show that trade liberalization reduces poverty and unemployment. The results on wages are ambiguous. Similarly, Bussolo, Lay, and Van der Mensbrugghe (2006) find that trade liberalization reduces poverty in Brazil. Also, Paquet and Savard (2009) analyze the effect of trade policies on the informal sector using a CGE model that includes an informal sector that can re-export products from the economy. They find that in Indonesia a reduction in import taxes produces an increase in the income of the informal sector. The authors argue that a decline in the exchange rate causes the income of the informal to expand.

Likewise, Cogneau and Robilliard (2008) study the impact of providing an agricultural subsidy, creating a part-time work program, and an untargeted transfer on the informal sector and poverty. All three policies decrease informal work and increase formal work, which leads to a decrease in poverty. Additionally, Stifel and Thorbecke (2003) build a CGE model with the informal sector to analyze the effect of trade liberalization in Africa on poverty in the formal and informal sectors. This paper finds that trade liberalization reduces poverty and inequality in the formal sector, but there is no effect in the informal sector. Also, Arguello et al. (2016) find that the mining and oil boom generated a fall in exports and production in other sectors of the economy. Equally, formal employment and skilled workers are favored, whereas there is no effect on the informal sector.

Furthermore, Botero and Correa (2018) examined the impact of three different changes in the tax system on informality and inequality in Colombia. Firstly, increasing the value-added tax by maintaining exceptions on some products leads to an increase in the informality rate and unemployment, and a decrease in inequality. Secondly, increasing the value-added tax without exceptions produces the same effects as the previous scenario. Finally, decreasing the corporate tax decreases informality, increases unemployment, and the inequality remains unchanged. In addition, Hernández (2011) shows that eliminating parafiscal taxes in Colombia and replacing them with other types of taxes in a general equilibrium context generates zero effects on the unemployment rate and non-significant changes in the informality rate.

Likewise, Atuesta and Hewings (2013) build a CGE model with microsimulations to evaluate the effect of drug legalization on unemployment and informality in Colombia. The authors argue that, when drugs are legalized, informality increases, and unemployment decreases. In addition, Rodríguez (2012) concludes that the elimination of parafiscal taxes and the introduction of new taxes on the income of corporations and high-skilled workers induces the creation of formal employment and the reduction of the unemployment rate. Finally, Osorio-Copete (2016) creates a stochastic CGE model to quantify the effects of the decrease in non-wage costs borne by the employer made in Colombia through tax reform in 2012. She finds that the reform reduced informality and accelerated the growth of formal employment.

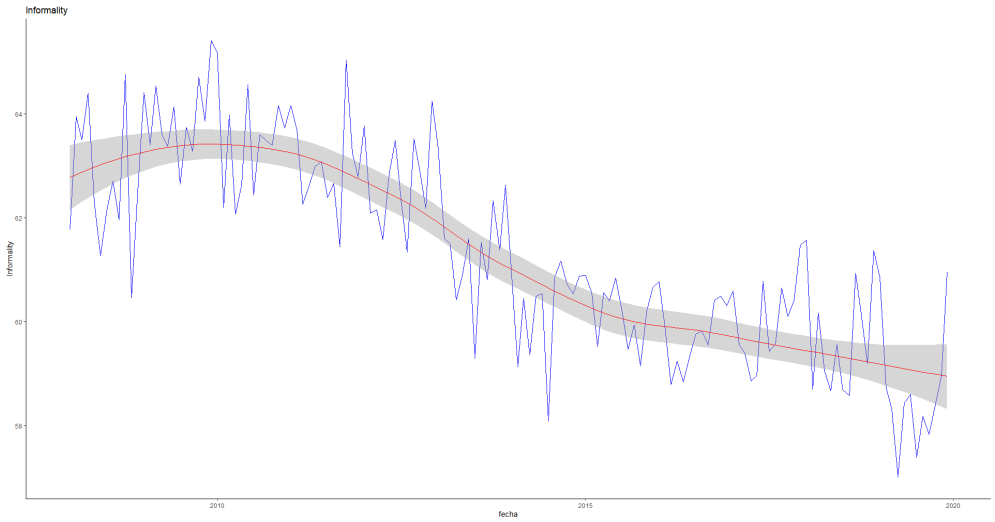
Given the above, existing studies on the effects of temporary employment on the economy have focused mainly on analyzing labor market outcomes regarding wages, effort, unemployment rate, and turnover. However, these studies do not consider either informality or poverty. Some studies have also analyzed the effect of some policies on informality, poverty, and the labor market. In general, these studies focus on trade liberalization, tax reforms, the use of transfers, subsidies, and international trade shocks. Nevertheless, none of these papers model temporary employment as an alternative form of contracting. Therefore, this paper will attempt to bring these two approaches together by analyzing

the effect of improved working conditions for temporary workers on poverty, informality, and Colombia's labor market.

3 Data analysis of informality and temporary work

In this section, we will analyze Colombia's situation concerning temporary employment and informality. We will proceed as follows. Firstly, we will analyze the evolution of informality in Colombia at the monthly level to observe how it has behaved since 2008, the year in which the Gran Encuesta Integrada de Hogares (GEIH) began to be conducted. Secondly, based on data from several countries, Colombia will be compared at the international level regarding the proportion of temporary employment in total employment. This will help to get a picture of the magnitude of temporary employment in Colombia. In addition, the evolution of temporary employment at the monthly level since 2008 will be analyzed using three different measures of temporary employment calculated from the GEIH. Finally, the evolution of temporary employment and informality are examined simultaneously to provide some intuitions for the theoretical model.

Figure 1: Informality



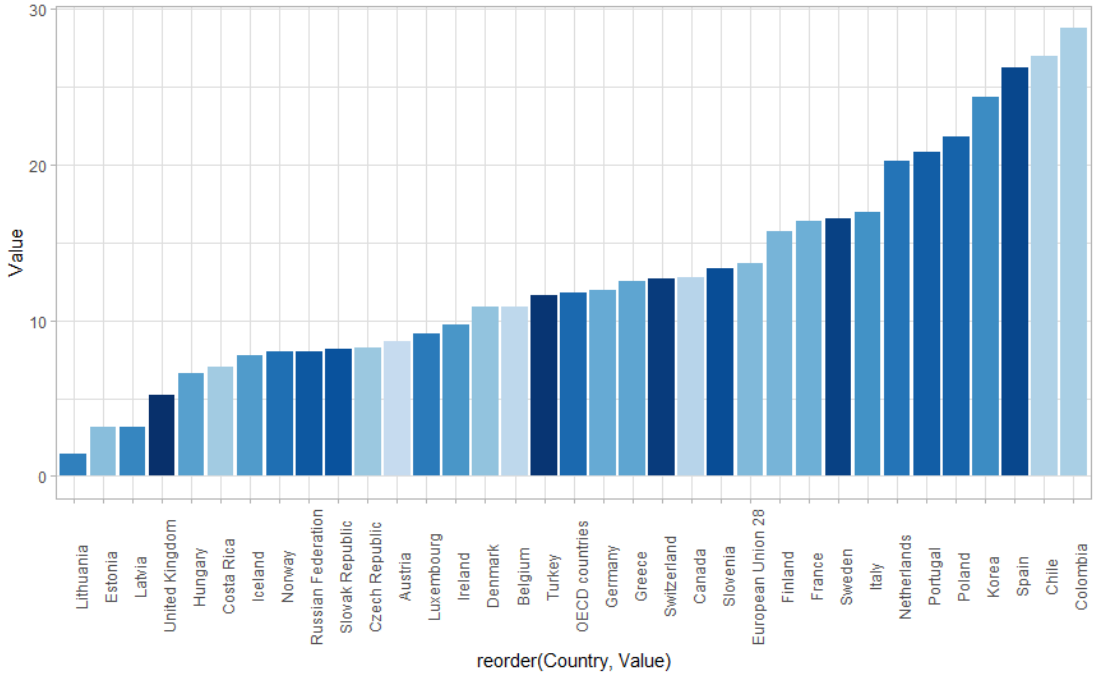
Source: Own elaboration based on DANE's Gran Encuesta Integrada de Hogares (GEIH).

The definition of informality used in this document is the one established by the Colombian National Administrative Department of Statistics (DANE)

based on the recommendations of the International Labor Organization (ILO). Thus, informal employment includes private employees and laborers working in enterprises with less than five workers, unpaid family workers, unpaid workers in enterprises of other households, domestic employees, day laborers, self-employed workers in establishments of up to five persons who are not professionals, and employers in enterprises with five workers or less. Laborers or government employees are not considered workers. Figure 1 shows the evolution of this measure between 2008 and 2019. The red line represents the result of a non-parametric regression to analyze the trend. It can be observed that informality had a slight increase between 2008 and 2010, but after this period it has been decreasing. However, the levels of informality are still of concern. In fact, in December 2019, the informality rate was 61%.

On the other hand, Figure 2 shows the percentage of temporary employment in 2019 relative to total dependent workers for several countries from the Organisation for Economic Co-operation and Development (OECD) database. According to these data, Colombia has one of the highest levels of temporary employment compared to the rest of the countries. As a matter of fact, in 2019 the temporary employment percentage was 28.9%, whereas the OECD average was 11.8%. Likewise, in Colombia, that figure for young people between the ages of 15 and 24 amounts to 41.12%. Therefore, temporary employment is unusually high in Colombia. These unusually high levels of temporary employment in Colombia become a problematic aspect considering that Colombia has a high rate of informality.

Figure 2: Temporary employment as a percentage of dependent employment.

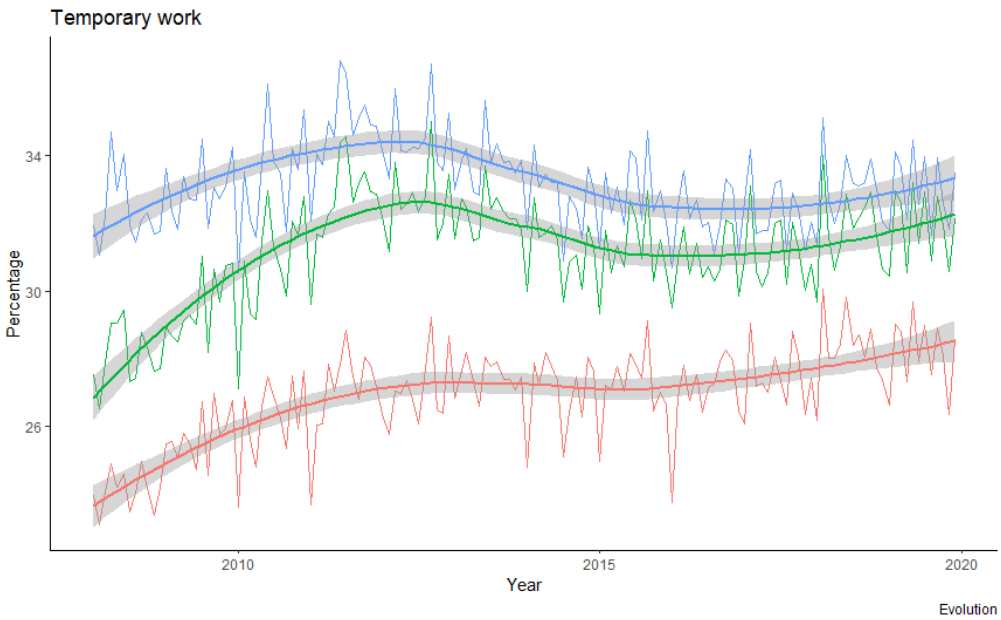


Source: Own elaboration based on OECD data, 2019.

Additionally, three different measures of temporary employment were calculated using the GEIH. The first measure refers to the Percentage of employees with a fixed-Term contract (PFT). The second is the Percentage of employees with a fixed-Term contract or Service provision contract (PFS). Finally, the Percentage of employees with a fixed-term contract, service provision contract, and contracts for a specific project or service (PFSP). Besides, these contracts include arrangements that are both labor and commercial in nature. The denominator used for these calculations was the total number of employees in the formal sector. The monthly evolution of these three indicators between the years 2008 and 2019 is shown in Figure 3.

Figure 3 shows that the three forms of measurement follow the same trend. It can be seen that temporary employment rises with a reasonably steep slope from 2008 to approximately 2013, and then this trend becomes a little less pronounced. In December 2019, the temporary employment rates were 28.6%, 32.1%, and 33.5% for the PFT, PFS, and PSFP, respectively. This, together with an informality level of 61%, means that the Colombian labor market generates low-quality jobs. The high levels of temporary employment in developing countries like Colombia have different implications than in more developed countries since temporary jobs mean instability, less training, and lower wages. In fact, in December 2019, 33% of the temporary workers considered their job unstable in Colombia.

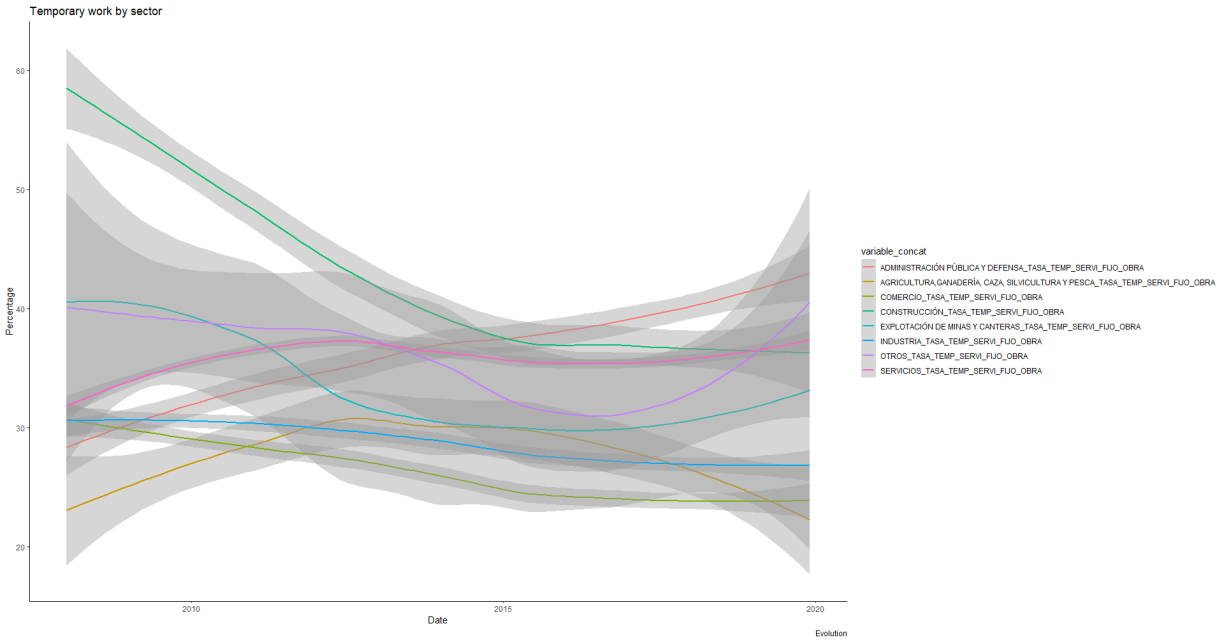
Figure 3: Temporary work



Source: Own elaboration, based on DANE's Gran Encuesta Integrada de Hogares (GEIH)

Alternatively, Figure 3.1 shows the monthly trend of temporary employment by sector measured as the PFSP. It is observed that the sectors showing an increasing trend in temporary employment during the entire period are Public Administration and Defense, services and the "Other" category. Moreover, the mining and quarrying sector shows an increase in the latter part of the period. By contrast, the construction, industry and commerce sectors show a stable trend at the end of the period. Finally, the agriculture, livestock, hunting, forestry, and fishing sector shows an upward trend in the most recent years studied. An interesting observation is that the percentages of temporary jobs remain unusually high in most sectors. For instance, en el 2019 for public administration and defense the temporary employment was 43%, for the category "Others" the figure was 50%, for construction and exploitation of mines and quarries this figure was 38%, and for the service sector, the percentage of temporary employment was 37%. The rest of the sectors presented values in the range of 24% and 25%, higher than the OECD countries' average.

Figure 3.1: Temporary employment trend by sector



Source: Own elaboration, based on DANE's Gran Encuesta Integrada de Hogares (GEIH)

In the main, even though informality in Colombia has shown a decreasing trend in recent years, the percentage of informal employment remains very high. On the contrary, temporary employment shows a growing trend regardless of the type of measure used, reaching unusually high levels compared to other countries. Likewise, although some sectors show stability or decrease in recent years in the indicator, the levels of temporary employment are still high. In conclusion, if the aggregate indicators are taken into account, informality has shown a decreasing trend whereas temporary employment has increased, which may mean that part of the formalization carried out in recent years may be driven to some extent by the increase in formal temporary employment.

4 Model Background

Macroeconomic models usually examine the impact of policies, shocks, and reforms on economic aggregates by considering the general equilibrium effects on the economy. For example, suppose a country needs to assess the effect of eliminating production taxes on the economy. In that case, it will certainly have to consider the subsequent fiscal deficit and include in the analysis the imposition of alternative taxes to raise the necessary funds to cover the deficit, given the

changes in all relative prices in the economy. However, these models do not allow for the examination of welfare changes at the individual level. Partial equilibrium microeconomic models do allow for this. Consequently, these models provide useful information for prioritizing and targeting policies to specific groups of the population but cannot consider general equilibrium effects. In this case, to analyze the impact of variations in firing costs on the labor market and poverty in the presence of informality, it is necessary to include both general equilibrium effects and variations in the poverty status of each individual.

Macro-micro simulation models consider heterogeneity at the individual level and the possible general equilibrium effects of a reform or an exogenous shock under study. These models consist of two modules. The macro module usually consists of a Computable General Equilibrium Model (CGE) that captures the dynamics of the aggregate variables and the economy's relative prices. In contrast, the micro module captures the results around individual changes. There are two ways of modeling this part. The first is to use a non-parametric model, in which individuals change state randomly, without a rational choice. The second is to use a behavioral model in which individuals make decisions based on wages and other variables. In this paper, the micro part will be approached with a behavioral model because the decisions of individuals in the face of changes in the wage vectors of temporary, permanent, and informal jobs are considered relevant. It should also be considered that, depending on the policy evaluation, more emphasis is placed on one module than the other. Since the interest is in examining the impact of a llabor reform, emphasis will be placed on the labor market in the macro module. The following sections describe the modules that will be used in this document.

5 The macroeconomic model Background

In the macro module, a recursive dynamic CGE model similar in spirit to the work of Decaluwé et al. (2003) will be used. However, significant changes will be considered in this paper in terms of the introduction of informality and temporary and permanent employment. We consider heterogeneous workers in the sense that there is a probability of job shirk. From the production and labor demand side, we include some changes. First, we introduce firing costs to be paid by the firm when laying off permanent workers. Moreover, we endogenize the probability of workers making an effort. Finally, there is an informal market that only uses labor to produce, does not pay taxes, and does not export goods and services. On the labor supply side, the worker decides whether to offer his labor to the formal or informal sector. If he offers his labor to the informal sector, he gets a job immediately since to work as an informal worker is enough to take a product and start selling it on the street. However, if the worker decides to offer his labor in the formal sector, he plays a lottery to see if he

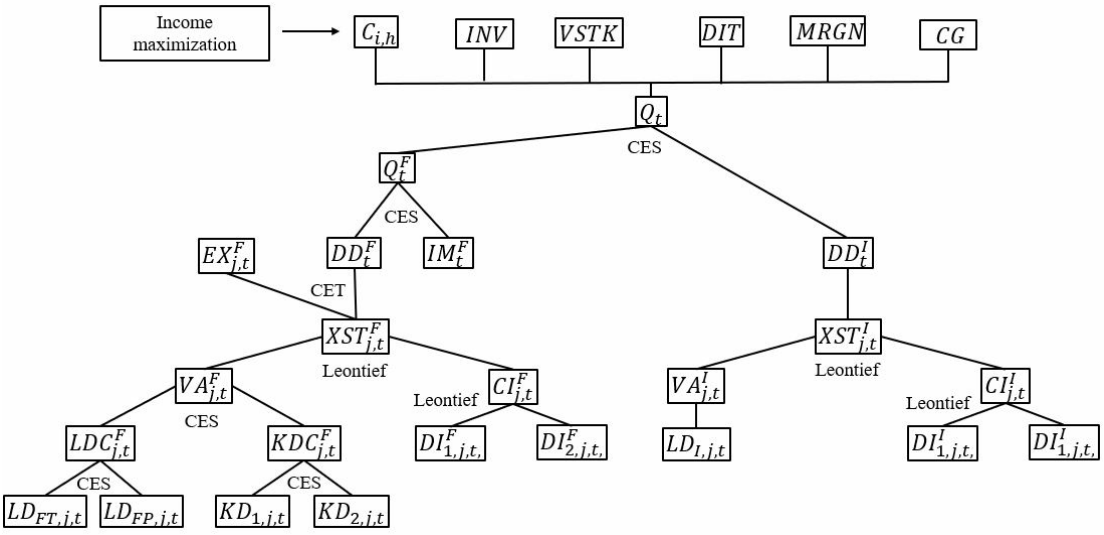
gets a job with a certain probability of becoming unemployed. If the worker gets the job, then he/she plays a lottery once again to see if he/she will be a temporary or permanent employee. In general, workers decide whether to be informal or formal, which is in line with recent literature (Bosch & Esteban-Pretel 2012). However, given the literature review findings that temporary jobs are not desirable, it is assumed that the worker does not decide whether to be temporary or permanent, but that this is a firm's decision. The following sections describe the modules that will be used in this document. The following section will describe the general structure of the model. Once this is done, another section will show the labor market details of the model.

5.1 Macro model overview

This part will describe the general structure of the model. For this purpose, a scheme of the economic structure will be used. In this sense, in the upper left part of Figure 5, it is observed that the formal composite labor ($LDC^F_{j,t}$) is formed by the optimal demands of temporary ($LD^F_{te,j,t}$) and permanent ($LD^F_{pe,j,t}$) labor, which result from the firm's cost minimization problem taking into account the effort and firing costs (this will be explained in more detail later). It is also observed that there is a composite capital ($KDC^F_{j,t}$) resulting from the combination of different types of capital i ($KD_{i,j,t}$). Formal composite labor and composite capital are combined through a CES function to generate formal value-added ($VA^F_{j,t}$). Likewise, formal aggregate intermediate consumption ($CI^F_{j,t}$) results from combining different types of formal intermediate products ($DI^F_{j,t}$) through a Leontief function. Then, formal value-added and formal aggregate intermediate consumption generate total formal production ($XST^F_{j,t}$), and this, in turn, is exported ($EX^F_{j,t}$) or sold on the domestic market (DD^F_t). What is sold in the domestic market, together with imports (IM^F_t), are aggregated through a CES to obtain the quantity demanded of the formal composite good (Q^F_t).

On the other hand, in the lower right part of Figure 4 is the structure of the informal sector. Informal value-added ($VA^I_{j,t}$) is produced using only informal labor demand. Likewise, informal intermediate aggregate consumption ($CI^I_{j,t}$) is produced through a Leontief function using different informal intermediate products ($DI^I_{j,t}$) as input. The total output of the informal sector ($XST^I_{j,t}$) is obtained by combining informal aggregate intermediate consumption and informal value-added through a Leontief function. The total output of the informal sector is not exported, and there are no informal imports, so the informal products sold and purchased (DD^I_t) in the local market correspond to those produced within the country.

Figure 4: Macro model structure



Source: Own elaboration

Furthermore, the quantity demanded of the formal composite good, and the quantity demanded of the informal good is added to obtain the quantity demanded of the economy's composite good. Finally, all the demands of the economy buy that composite good. These demands refer to consumption ($C_{i,h}$), investment (INV), change in stocks ($VSTK$), demand for intermediate goods (DIT), demand for goods such as trade or transport margin ($MRGN$), and demand by the government (CG). Special attention is given to the demand for consumption since before the household decides how much to consume, it must first maximize its income (this will be explained later). In this document, the explanation will be focused on the aspects of the labor market due to the objectives of the research. However, the equations of the entire model can be consulted in detail in Appendix 1 at the end of this document. The following section will explain the labor market in the model.

5.2 The labor market in the Model

5.2.1 Households

In a typical computable general equilibrium model, household income is given, and households only make decisions around their consumption to maximize their utility. However, in our model, household income will be determined by choosing the proportion of formal and informal workers in the household. Therefore, these decisions are modeled in two stages. First, the household decides what

proportion of its workers it allocates to formal work and what proportion it allocates to informal work. Once the household has made the above decisions, it maximizes its utility. These stages can be summarized: i) income maximization; ii) utility maximization. This specification implies Hicksian separability. Likewise, in line with recent literature on informality, workers choose between being informal or formal in the model.

Note that the household does not decide the proportion of workers it assigns to temporary or permanent work. This decision is made by the company and will be explained below . In addition, heterogeneity is assumed among workers in that there are some who work hard and others who do not. Hence, the income of households also depends on the rate at which they exert themselves because if they do not, they are fired with a certain probability. This section is organized into two parts. First, we micro-found the effort rates of each type of worker. Finally, we analyze the household's decision concerning the allocation between formal and informal employment.

Effort rates A shirking worker can be found out and be fired with a certain probability and stop receiving income, so it is crucial to make this decision explicit through its micro-foundation. Accordingly, this will be done for both temporary and permanent workers.

Effort rate for temporary workers We first derive the effort function for temporary workers. Thus, the temporary worker's earnings are posed for the scenario in which he or she evades and in which he or she does not evade. If the temporary worker does not shirk, he/she gains the following

$$V_{TN}^{h} = \ln W_{te,j,t}^{F} - b + \ln\left(1 - \varepsilon_{te}\right) + \mu_{TN}^{h} \tag{1}$$

Where where V_{TN}^{h} is the net compensation of a temporary worker who does not shirk, $W_{te,j,t}^{F}$ is the temporary worker's wage in industry j, ε_{te} is the temporary worker's effort level, μ_{TN}^{h} is a idiosyncraticvariable and b is the non-wage cost of being a temporary worker due to uncertainty and other disadvantages . The net compensation of a temporary worker who does not shirk depends positively on the temporary wage and negatively on the effort he/she makes. Furthermore, b is negatively related to the worker's net earnings. On the other hand, the utility of a worker who shirks is

$$V_{TS}^{h} = \pi\left(\ln W_{l,t}^{I} - a\right) + (1 - \pi)\left(\ln W_{te,j,t}^{F} - b\right) + \mu_{TS}^{h}, \qquad a > b \tag{2}$$

Where where V_{TS}^h is the net compensation of a temporary shirking worker, $W_{l,t}^I$ is the informal worker's wage in industry j, π is the probability of being caught shirking, μ_{TS}^h is a idiosyncraticvariable and a is the non-wage cost of being a informal worker due to uncertainty and other disadvantages. The net compensation of a temporary worker who shirks is a weighted average, where the weights are the probability of being caught (π) and the probability of not being caught ($1-\pi$). If the worker is captured and therefore fired, then the worker must go to the informal sector and earn the informal wage, but suffer the consequences of filling an informal vacancy. In this scenario the net gain of the temporary worker would be $\ln W_{l,t}^I - a$. If the worker is not discovered, then the temporary worker's earnings would depend on the temporary wage and the non-wage costs of being a temporary worker ($\ln W_{te,j,t}^F - b$), but without experiencing disutility from the effort ($1-\varepsilon_{te}^F$).

Given the above, for a worker to decide to shirk, the net compensation for shirking must be greater than that obtained by not shirking, that is

$$V_{TS}^h \geq V_{TN}^h \tag{3}$$

Using Equations (1) and (2), Equation (3) implies

$$\mu_{TS}^h - \mu_{TN}^h \geq \pi a + \pi \ln \frac{W_{te,j,t}^F}{W_{l,t}^I}\left(1-\varepsilon_{te}^F\right) - \pi b$$

Assuming that μ_{TS}^h and μ_{TN}^h follow a Weibull distribution, the difference between these two distributions implies that

$$\mu_{TS}^h - \mu_{TN}^h \geq \frac{1}{1+\frac{1}{\left(1-\varepsilon_{te}^F\right)\delta_T}\frac{\left(W_{l,t}^I\right)^{\pi}}{\left(W_{te,j,t}^F\right)^{\pi}}}$$

Let $\delta_{te} = e^{\pi(a-b)}$, then the probability that a temporary worker drawn randomly from the employed population chooses not to shirk is

$$f_{te,j,t}^F = \frac{1}{1+\frac{1}{\left(1-\varepsilon_{te}^F\right)\delta_T}\left(WR_{te,j,t}\right)^{-\pi}} \tag{4}$$

Where $WR_{te,j,t}$ is the relative wage of a temporary worker with respect to the informal wage ($\frac{W_{te,j,t}^F}{W_{l,t}^I}$). It is observed that the probability of not shirking

depends negatively on the effort of temporary workers. Intuitively, if the temporary worker puts more effort than he/she should into his job, he is more likely to plan to shirk at some point, given the disutility this generates. Additionally, if the temporary worker's relative wage increases, this probability increases. This result is in line with the efficiency wage theory, which holds that workers exert more effort when assigned an optimal wage that motivates them to perform their tasks. Finally, when δ_{te} increases so does the effort probability. This means that the higher the non-wage cost differential (a-b) between informal and temporary jobs, the higher the effort probability. Hence, the non-wage costs of being informal act as a threat that causes temporary workers not to shirk.

Effort rate for permanent workers To obtain the effort rate for permanent workers, the procedure will be similar to that used for temporary workers. The net compensation of a permanent worker who does not elude is

$$V_{PN}^{h} = \ln W_{pe,j,t}^{F} + b + \ln\left(1 - \varepsilon_{pe}^{F}\right) + \mu_{PN}^{h} \tag{5}$$

Where where V_{PN}^{h} is the net compensation of a permanent worker who does not shirk, $W_{te,j,t}^{F}$ is the permanent worker's wage in industry j, ε_{pe}^{F} is the permanent worker's effort level, μ_{PN}^{h} is a idiosyncraticvariable and b is the non-wage benefit of being a permanent worker due to to stability and other disadvantages . The net compensation of a permanent worker who does not shirk depends positively on his/her wage and negatively on the effort he/she makes. Besides, the net compensation of being a shirking permanent worker is

$$V_{PS}^{h} = \pi\left(\ln W_{l,t}^{I} - a\right) + (1 - \pi)\left(\ln W_{pe,j,t}^{F} + b\right) + \mu_{PS}^{h} \tag{6}$$

Where where V_{TS}^{h} is the utility of a permanent shirking worker and μ_{PS}^{h} is a idiosyncratic variable. Similarly, the net compensation of a permanent worker who shirks is a weighted average, where the weights are the probability of being caught (π) and the probability of not being caught ($1 - \pi$). If the permanent worker is captured shirking and therefore fired, then the worker must go to the informal sector and earn the informal wage, but suffer the consequences of filling an informal vacancy. Therefore, the net gain of the permanent worker would be $\ln W_{l,t}^{I} - a$. In contrast, if the permanent worker is not caught, then the worker's earnings would depend on the permanent wage and the non-wage benefit of being a permanent worker ($\ln W_{pe,j,t}^{F} + b$), but without experiencing disutility from the effort ($\ln 1 - \varepsilon_{pe}^{F}$).

In order for a permanent employee to shirk his/her responsibilities, the following must be met

$$V_{PS}^{h} \geq V_{PN}^{h} \tag{7}$$

Using Equations (5) and (6), Equation (7) implies

$$\mu_{PS}^{h} - \mu_{PN}^{h} \geq \ln\left(\frac{W_{pe,j,t}^{F}}{W_{l,t}^{I}}\right)^{\pi}\left(1-\varepsilon_{pe}^{F}\right) + \pi(b+a)$$

Se asume que μ^h sigue una districución gumbel, la diferencia entre estas dos distribuciones es una función logit

$$\mu_{PS}^{h} - \mu_{PN}^{h} \geq \frac{1}{1+\frac{1}{\left(1-\varepsilon_{pe}^{F}\right)e^{\pi(b+a)}}\frac{\left(W_{l,t}^{I}\right)^{\pi}}{\left(W_{pe,j,t}^{F}\right)^{\pi}}}$$

Let $\delta_{pe} = e^{\pi(a+b)}$, then the probability that a permanent worker drawn randomly from the employed population chooses not to shirk is

$$f_{pe,j,t}^{F} = \frac{1}{1+\frac{1}{\left(1-\varepsilon_{pe}^{F}\right)\delta_{pe}}\left(WR_{pe,j,t}\right)^{-\pi}}$$

Where $WR_{pe,j,t}$ is the relative wage of a permanent worker with respect to the informal wage ($\frac{W_{pe,j,t}^{F}}{W_{l,t}^{I}}$). The probability of no shirk is increasing in relative wage and decreasing in the effort. Likewise, F depends positively on delta_P. This means that the higher a and b are, the higher the probability that the permanent worker will exert effort.

Elasticity of f To generalize the above, we use the index l to denote the type of workers. There are two types of workers, permanent and transitory workers. If the workers are permanent workers $l = pe$, and if they are temporary workers $m = te$. Thus, the percentage of workers exerting effort in each type of workers is denoted by f_{Fm} and is equal to:

$$f_{l,j,t}^{F} = \frac{1}{1+\frac{1}{(1-\varepsilon_{l})\delta_{l}}\left(\frac{W_{l,j,t}^{F}}{W_{l,t}^{I}}\right)^{-\pi}},\ l = \{pe, te\}.$$

Similarly, the relative wage of formal workers with respect to informal workers as

$$WR_{l,j,t} = \frac{W^F_{l,j,t}}{W^I_{l,t}},\ m = \{pe, te\}.$$

Hence

$$f^F_{l,j,t} = \frac{1}{1 + \frac{1}{(1-\varepsilon_l)\delta_l}\left(WR_{l,j,t}\right)^{-\pi}},\ m = \{pe, te\}. \tag{8}$$

Now, to find the elasticity of f with respect to the wage, we proceed as follows. First, we take the logarithm of (8)

$$\ln\left(f^F_{l,j,t}\right) = \ln\left(\frac{1}{1 + \frac{1}{(1-\varepsilon_l)\delta_l}\left(WR_{l,j,t}\right)^{-\pi}}\right) = \ln(1) - \ln\left(1 + \frac{1}{(1-\varepsilon_l)\,\delta_l}\left(WR_{l,j,t}\right)^{-\pi}\right)$$

Thus,

$$\ln(f^F_{l,j,t}) = -\ln\left(1 + \frac{1}{(1-\varepsilon_l)\,\delta_l}\left(WR_{l,j,t}\right)^{-\pi}\right)$$

Then, deriving $\ln(f^F_{l,j,t})$ with respect to $W^F_{l,j,t}$ we get

$$\frac{\partial f^F_{l,j,t}}{\partial W^F_{l,j,t}} = \pi\frac{\left(f^F_{l,j,t}\right)\left(1 - f^F_{l,j,t}\right)}{W^F_{l,j,t}}$$

Dividing this by $f^F_{l,j,t}$ gives

$$\frac{\partial f^F_{l,j,t}}{\partial W^F_{l,j,t}}\frac{W^F_{l,j,t}}{f^F_{l,j,t}} = \pi\left(1 - f^F_{l,j,t}\right) \tag{9}$$

The elasticity of f with respect to the wage is positive if and only if $\left(1 - f^F_{l,j,t}\right) > 0$.

Income maximization Now that the effort rates are known, the household income maximization problem can be analyzed. Household income ($YHL_{h,t}$) depends on the proportion of workers employed in both formal ($LS_{h,F,t}$) and informal ($LS_{h,I,t}$) jobs. The household makes such a decision subject to a Constant Elasticity of Transformation (CET) function. Thus, it is assumed that these types of work are not perfect substitutes and that an optimal combination of both types of work must be selected.

Before setting out the household's program, let us define

$$LS_{I,t} = \sum_{h} LS_{h,I,t}$$

$$LS^{F}_{ls\_ft,t} = \sum_{h} LS_{h,F,t}$$

On the other hand, the household must consider that the formal sector wage is uncertain because it is subject to the probability of obtaining formal employment and the probability of obtaining a temporary or permanent job. Hence, the household maximizes its income based on the expected wage in the formal sector $\left(W^{e}_{F,t}\right)$

$$\begin{aligned} W^{e}_{F,t} &= \left(PU^{F}_{t} \times 0\right) \\ &+ \left(1 - PU^{F}_{t}\right) \sum_{j} \Omega^{F}_{j,t} \left[\Phi^{F}_{j,t} \left(\left(\pi f^{F}_{te,j,t} + 1 - \pi\right) W^{F}_{te,j,t} + c^{F}_{te} \pi \left(1 - f^{F}_{te,j,t}\right)\right) + \left(1 - \Phi^{F}_{j,t}\right) \left(\left(\pi f^{F}_{pe,j,t} + 1 - \pi\right) W^{F}_{pe,j,t} + c^{F}_{pe} \pi \left(1 - f^{F}_{pe,j,t}\right)\right)\right] \end{aligned}$$

The expected wage in the formal sector is a weighted average between what the worker receives when he/she does not find a job (nothing) and what he/she receives when is hired. The probability of not finding a job is PU^{F}_{t}, which is defined as the ratio of the number of unemployed to the total formal labor supply $\left(\frac{\mu_t}{LS^{F}_{ls\_ft,t}}\right)$. If the worker is hired, he receives a weighted average between the wage of temporary jobs and the wage of permanent jobs. Likewise, the probability of finding a job in industry j is $\Omega^{F}_{j,t}$, which is the ratio between the demand for labor in industry j and the total labor demand for labor in all industries $\left(\frac{LDC^{F}_{j,t}}{\sum_j LDC^{F}_{j,t}}\right)$. The probability of finding a temporary job in the industri j is $\Phi^{F}_{j,t}$, which is defined as the division between the demand for temporary labor and the total demand for labor in the formal sector $\left(\frac{\sum_j LD_{FT,,j,t}}{\sum_j \sum_m LD_{Fm,,j,t}}\right)$. Moreover, if the job found is temporary, the probability of effort weights its income. The proportion of labor that the household allocates to temporary work that exerts effort gets $f^{F}_{te,j,t} W^{F}_{te,j,t}$, whereas the proportion of temporary labor allocated that does not exert effort gets $(1-\pi)\left(1 - f^{F}_{te,j,t}\right) W^{F}_{te,j,t}$. Note that the household receives $\left(1 - f^{F}_{te,j,t}\right) W^{F}_{te,j,t}$ only if it is not caught shirking,

hence it is multiplied by $(1-\pi)$. Thus, the representative household solves the programme

$$\underset{LS_{h,I,t},LS_{h,F,t}}{Max} \quad YHL_{h,t} = \; W^{I}_{l,t} LS_{h,I,t} + W^{e}_{F,t} LS_{h,F,t}$$

Or equivalently

$$\underset{LS_{h,I,t},LS_{h,F,t}}{Max} \; W^{I}_{l,t} LS_{h,I,t} + \left(1 - PU^{F}_{t}\right) \sum_{j} \Omega^{F}_{j,t} \left[\Phi^{F}_{j,t} \left(\left(\pi f^{F}_{te,j,t} + 1 - \pi\right) W^{F}_{te,j,t} + c^{F}_{te} \pi \left(1 - f^{F}_{te,j,t}\right)\right) + \left(1 - \Phi^{F}_{j,t}\right) \left(\left(\pi f^{F}_{pe,j,t} + 1 - \pi\right) W^{F}_{pe,j,t} + c^{F}_{pe} \pi \left(1 - f^{F}_{pe,j,t}\right)\right)\right]$$

Subject to

$$LS_{h,t} = \left(\psi_{I} \left(LS_{h,I,t}\right)^{\frac{1+\sigma^{LS}}{\sigma^{LS}}} + \psi_{F} \left(LS_{h,F,t}\right)^{\frac{1+\sigma^{LS}}{\sigma^{LS}}}\right)^{\frac{\sigma^{LS}}{1+\sigma^{LS}}}$$

Where $LS_{h,I,t}$ is the informal labor supply and $LS_{h,F,t}$ is the formal labor supply. Also, $W^{I}_{l,t} LS_{h,I,t}$ is the income received by the household for workers it assigns to informal work and $W^{e}_{F,t} LS_{h,F,t}$ is the income received by the household due to the workers assigned to formal jobs. Furthermore, ψ_{I} and ψ_{F} are the CET share parameters corresponding to informal and formal work, respectively. Then, Lagrangian for the maximization problem is

$$\begin{aligned}
\mathcal{L} \; &= W^{I}_{l,t} LS_{h,I,t} \\
&+ \left(1 - PU^{F}_{t}\right) \sum_{j} \Omega^{F}_{j,t} \left[\Phi^{F}_{j,t} \left(\left(\pi f^{F}_{te,j,t} + 1 - \pi\right) W^{F}_{te,j,t} + c^{F}_{te} \pi \left(1 - f^{F}_{te,j,t}\right)\right) + \left(1 - \Phi^{F}_{j,t}\right) \left(\left(\pi f^{F}_{pe,j,t} + 1 - \pi\right) W^{F}_{pe,j,t} + c^{F}_{pe} \pi \left(1 - f^{F}_{pe,j,t}\right)\right)\right] LS_{h,F,t} \\
&- \lambda \left[\left(\psi_{I} \left(LS_{h,I,t}\right)^{\frac{1+\sigma^{LS}}{\sigma^{LS}}} + \psi_{F} \left(LS_{h,F,t}\right)^{\frac{1+\sigma^{LS}}{\sigma^{LS}}}\right)^{\frac{\sigma^{LS}}{1+\sigma^{LS}}} - LS_{h,t}\right]
\end{aligned}$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial LS_{h,I,t}} = 0 \Leftrightarrow$$

$$W^{I}_{l,t} - \lambda \left[\left(\psi_{I} \left(LS_{h,I,t}\right)^{\frac{1+\sigma^{LS}}{\sigma^{LS}}} + \psi_{F} \left(LS_{h,F,t}\right)^{\frac{1+\sigma^{LS}}{\sigma^{LS}}}\right)^{-\frac{1}{1+\sigma^{LS}}} \psi_{I} \left(LS_{h,I,t}\right)^{\frac{1}{\sigma^{LS}}}\right] = 0 \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial LS_{h,F,t}} = 0 \Rightarrow$$

$$\left(1-PU_t^F\right)\sum_j \Omega_{j,t}^F\left[\Phi_{j,t}^F\left(\left(\pi f_{te,j,t}^F+1-\pi\right)W_{te,j,t}^F+c_{te}^F\pi\left(1-f_{te,j,t}^F\right)\right)+\left(1-\Phi_{j,t}^F\right)\left(\left(\pi f_{pe,j,t}^F+1-\pi\right)W_{pe,j,t}^F+c_{pe}^F\pi\left(1-f_{pe,j,t}^F\right)\right)\right]$$
$$=\lambda\left[\left(\psi_I\left(LS_{h,I,t}\right)^{\frac{1+\sigma^{LS}}{\sigma^{LS}}}+\psi_F\left(LS_{h,F,t}\right)^{\frac{1+\sigma^{LS}}{\sigma^{LS}}}\right)^{-\frac{1}{1+\sigma^{LS}}}\psi_F\left(LS_{h,F,t}\right)^{\frac{1}{\sigma^{LS}}}\right] \tag{11}$$

$$\frac{\partial\mathcal{L}}{\partial\lambda}=\left(\psi_I\left(LS_{h,I,t}\right)^{\frac{1+\sigma^{LS}}{\sigma^{LS}}}+\psi_F\left(LS_{h,F,t}\right)^{\frac{1+\sigma^{LS}}{\sigma^{LS}}}\right)^{\frac{\sigma^{LS}}{1+\sigma^{LS}}}-LS_{h,t}=0 \tag{12}$$

Dividing (10)/(11) we found the relative supplies

$$LS_{h,I,t}=\left(\frac{\psi_F}{\psi_I}\right)^{\sigma^{LS}}\left(\frac{W_{l,t}^I}{\left(1-PU_t^F\right)\sum_j\Omega_{j,t}^F\left[\Phi_{j,t}^F\left(\left(\pi f_{te,j,t}^F+1-\pi\right)W_{te,j,t}^F+c_{te}^F\pi\left(1-f_{te,j,t}^F\right)\right)+\left(1-\Phi_{j,t}^F\right)\left(\left(\pi f_{pe,j,t}^F+1-\pi\right)W_{pe,j,t}^F+c_{pe}^F\pi\left(1-f_{pe,j,t}^F\right)\right)\right]}\right)^{\sigma^{LS}}LS_{h,F,t} \tag{13}$$

To find the formal job supply we replace (13) in (12)

$$LS_{h,F,t}=\left(\frac{\left(\psi_I\right)^{\sigma^{LS}}\left(W_{F,t}^e\right)^{1+\sigma^{LS}}}{\left(\psi_F\right)^{1+\sigma^{LS}}\left(W_{l,t}^I\right)^{1+\sigma^{LS}}+\psi_F\left(\psi_I\right)^{\sigma^{LS}}\left(W_{F,t}^e\right)^{1+\sigma^{LS}}}\right)^{\frac{\sigma^{LS}}{1+\sigma^{LS}}}$$

Or equivalent

$$LS_{h,F,t}=\left(\frac{1}{\psi_F}\frac{W_t^{e^F}}{\widehat{W}}\right)^{\sigma^{LS}}LS_{h,t} \tag{14}$$

Where

$$\widehat{W}=\left(\psi_I\left(\frac{W_{l,t}^I}{\psi_I}\right)^{1+\sigma^{LS}}+\psi_F\left(\frac{W_{F,t}^e}{\psi_F}\right)^{1+\sigma^{LS}}\right)^{\frac{1}{1+\sigma^{LS}}}$$

The formal job supply depends positively on the employment rate and the weighted average between the wage of temporary workers and formal workers. It should be noted that generally, the wage of temporary workers in Colombia is

lower than for formal workers. Therefore if the proportion of those employed in temporary jobs with respect to the total number of employed increases $\left(\Phi_{j,t}^{F}\right)$, the weighted average will decrease, negatively affecting the formal labor supply. This is quite interesting because it captures the intuition that high levels of temporary jobs in the formal sector decrease formal labor supply. Also, it can be observed that the informal sector wage negatively affects the formal labor supply.

On the other hand, to find the informal labor supply we replace (14) in (12) to get

$$LS_{h,I,t} = \left(\frac{1}{\psi_I}\frac{W_{l,t}^{I}}{\widehat{W}}\right)^{\sigma^{LS}} LS_{h,t} \tag{15}$$

The informal labor supply depends positively on the informal wage. On the other hand, the informal labor supply depends negatively on the employment rate and the weighted average between temporary workers' wages and those of formal workers.

Utility maximization When the household chooses its optimal combination of temporary and permanent labor, it decides how much to consume, subject to a budget constraint in order to maximize its utility. A Stone-Geary utility function is assumed. The intuition behind this utility function is that the household first secures its subsistence consumption and the remaining budget is allocated to other goods according to its preferences. The utility function is

$$U_{h,t} = \prod_i \left(C_{i,h,t} - C_{i,h,t}^{MIN}\right)^{\gamma_{i,h}^{LES}} \quad \text{where} \quad \sum_i \gamma_{i,h}^{LES} = 1$$

This is equivalent to

$$\ln U_{h,t} = \sum_i \gamma_{i,h}^{LES} \ln\left(C_{i,h,t} - C_{i,h,t}^{MIN}\right)$$

The budget constraint is:

$$\sum_i PC_{i,t} C_{i,h,t} = CTH_{h,t}$$

Thus, the Lagrangian is

$$\mathcal{L}_h = \sum_i \gamma_{i,h}^{LES} \ln\left(C_{i,h,t} - C_{i,h,t}^{MIN}\right) - \lambda\left(\sum_i PC_{i,t}C_{i,h,t} - CTH_{h,t}\right)$$

The first-order conditions are

$$\frac{\partial \mathcal{L}_h}{\partial \lambda} = \sum_i PC_{i,t}C_{i,h,t} - CTH_{h,t} = 0 \tag{16}$$

$$\frac{\partial \mathcal{L}_h}{\partial C_{i,h,t}} = \gamma_{i,h}^{LES}\frac{1}{C_{i,h,t} - C_{i,h,t}^{MIN}} - \lambda PC_{i,t} = 0 \Rightarrow \lambda PC_{i,t}\left(C_{i,h,t} - C_{i,h,t}^{MIN}\right) = \gamma_{i,h}^{LES} \tag{17}$$

Suming (17) over i and remembering that $\sum_i \gamma_{i,h}^{LES} = 1$ we obtain

$$\lambda\sum_i PC_{i,t}\left(C_{i,h,t} - C_{i,h,t}^{MIN}\right) = \sum_i \gamma_{i,h}^{LES} = 1 \Rightarrow$$

$$\sum_i PC_{i,t}\left(C_{i,h,t} - C_{i,h,t}^{MIN}\right) = \frac{1}{\lambda}$$

From (16) we know that

$$\sum_i PC_{i,t}C_{i,h,t} = CTH_{h,t}$$

Hence

$$CTH_{h,t} - \sum_i PC_{i,t}C_{i,h,t}^{MIN} = \frac{1}{\lambda}$$

Replacing this in (17), we obtain the demand function

$$PC_{i,t}C_{i,h,t} = PC_{i,t}C_{i,h,t}^{MIN} + \gamma_{i,h}^{LES}\left(CTH_{h,t} - \sum_i PC_{i,t}C_{i,h,t}^{MIN}\right) \tag{18}$$

Where $C_{i,h,t}$ is the consumption of good i by household type h, $C_{i,h,t}^{MIN}$ is the minimum consumption of good i by household type h, $\gamma_{i,h}^{LES}$ is the marginal share of good i in the budget of household type h, and $CTH_{h,t}$ is the consumption budget of household type h.

5.2.2 Labor demand and wages

In the general equilibrium model, composite labor is input to value-added. Thus, the firm selects the labor composition that minimizes labor cost. In our context, the firm must select the optimal amount of permanent and temporary labor used in the form of composite labor in value-added. Moreover, we allow for heterogeneity among workers in terms of effort. Therefore, the firm selects efficiency wages to incentivize workers not to shirk since there is a probability that they will not exert effort. Consequently, wages will be above the equilibrium point, which leads to unemployment. Also, if the firm catches a shirking worker, it will have to pay a cost to fire him. The unit cost of a worker of type l (g_l) is

$$\begin{aligned} g^F_{l,j,t} &= \left[f^F_{l,j,t} + (1-\pi)\left(1 - f^F_{l,j,t}\right)\right] WTI^F_{l,j,t} + c^F_l \pi \left(1 - f^F_{l,j,t}\right) + E^F_{l,t} \\ g &= \left[1 - \pi + \pi f^F_{l,j,t}\right] WTI^F_{l,j,t} + c^F_l \pi \left(1 - f^F_{l,j,t}\right) + E^F_{l,t} \end{aligned} \tag{19}$$

With

$$WTI^F_{l,j,t} = \left(1 + ttiw^F_{l,j,t}\right) W^F_{l,j,t}$$

$$E^F_{pe,t} > E^F_{te,t} = 0$$

In equation (19), $f^F_{l,j,t} WTI^F_{l,j,t}$ are the wages paid to workers who exert effort (including payroll taxes $ttiw^F_{l,j,t}$), $(1-\pi)\left(1 - f^F_{l,j,t}\right) WTI^F_{l,j,t}$ are the wages paid to workers who do not exert effort and are not detected by the firm (including payroll taxes $ttiw^F_{l,j,t}$), $E^F_{l,t}$ is the cost of training an employee and $c^F_l \pi \left(1 - f^F_{l,j,t}\right) W^F_{l,j,t}$ are the costs of firing workers who do not exert effort and are detected by the firm, which are proportional to the wages. Therefore, the company minimizes its costs taking into account an aggregator function. Using the l-index, the minimization problem is expressed as:

$$\min \sum_l g^F_{l,j,t} LD^F_{l,j,t}$$

Subject to

$$LDC^F_{j,t} = B^{LDF}_j \left(\beta^{LD^F}_{pe,j} \left(A^F_{pe} \varepsilon^F_{pe} f^F_{pe,j,t} LD^F_{pe,j,t}\right)^{\frac{\sigma-1}{\sigma}} + \beta^{LD^F}_{te,j} \left(A^F_{te} \varepsilon^F_{te} f^F_{te,j,t} LD^F_{te,j,t}\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

With

$$A^F_{pe} > A^F_{te} = 1$$

In this program, $LDC^F_{j,t}$ represents composite work, $LD^F_{l,j,t}$ the demand for l-type work, A^F_l the productivity of l-type work, A^F_l the effort of l-type work and $f^F_{l,j,t}$ is the effort rate of l-type work. B^{LDF}_j is the scale parameter of the CES function, and $\beta^{LD^F}_{l,j}$ is the CES share parameter corresponding to $l-$type work. The Lagrangian is

$$\mathcal{L} = \sum_m g^F_{l,j,t} LD^F_{l,j,t} - \lambda \left[B^{LDF}_j \left(\beta^{LD^F}_{pe,j} \left(A^F_{pe} \varepsilon^F_{pe} f^F_{pe,j,t} LD^F_{pe,j,t} \right)^{\frac{\sigma-1}{\sigma}} + \beta^{LD^F}_{te,j} \left(A^F_{te} \varepsilon^F_{te} f^F_{te,j,t} LD^F_{te,j,t} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - LDC^F_{j,t} \right]$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial_m LD^F_{l,j,t}} = 0 \Leftrightarrow g^F_{l,j,t} = \lambda B^{LDF}_j \left(\beta^{LD^F}_{pe,j} \left(A^F_{pe} \varepsilon^F_{pe} f^F_{pe,j,t} LD^F_{pe,j,t} \right)^{\frac{\sigma-1}{\sigma}} + \beta^{LD^F}_{te,j} \left(A^F_{te} \varepsilon^F_{te} f^F_{te,j,t} LD^F_{te,j,t} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}-1} \beta^{LD^F}_{l,j} \left(A^F_l \varepsilon^F_l f^F_{l,j,t} \right)^{\frac{\sigma-1}{\sigma}} \left(LD^F_{l,j,t} \right)^{\frac{-1}{\sigma}} \tag{20}$$

$$\frac{\partial \mathcal{L}}{\partial W^F_{l,j,t}} = 0 \Leftrightarrow \frac{\partial g^F_{l,j,t}}{\partial W^F_{l,j,t}} LD^F_{l,j,t} = \lambda B^{LDF}_j \left(\beta^{LD^F}_{pe,j} \left(A^F_{pe} \varepsilon^F_{pe} f^F_{pe,j,t} LD^F_{pe,j,t} \right)^{\frac{\sigma-1}{\sigma}} + \beta^{LD^F}_{te,j} \left(A^F_{te} \varepsilon^F_{te} f^F_{te,j,t} LD^F_{te,j,t} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}-1} \beta^{LD^F}_{l,j} \left(A^F_l \varepsilon^F_l LD^F_{l,j,t} \right)^{\frac{\sigma-1}{\sigma}} \left(f^F_{l,j,t} \right)^{\frac{-1}{\sigma}} \frac{\partial f^F_{l,j,t}}{\partial W^F_{l,j,t}} \tag{21}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Leftrightarrow B^{LDF}_j \left(\beta^{LD^F}_{pe,j} \left(A^F_{pe} \varepsilon^F_{pe} f^F_{pe,j,t} LD^F_{pe,j,t} \right)^{\frac{\sigma-1}{\sigma}} + \beta^{LD^F}_{te,j} \left(A^F_{te} \varepsilon^F_{te} f^F_{te,j,t} LD^F_{te,j,t} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - LDC^F_{j,t} = 0 \tag{22}$$

We firts devire the labor demands. Using (20), demands for temporal and permanente workers are divided

$$LD^F_{te,j,t} = \left(\frac{\beta^{LD^F}_{te,j}}{\beta^{LD^F}_{pe,j}} \right)^{\sigma} \frac{\left(A^F_{te} \varepsilon^F_{te} f^F_{te,j,t} \right)^{\sigma-1}}{\left(A^F_{pe} \varepsilon^F_{pe} f^F_{pe,j,t} \right)^{\sigma-1}} \left(\frac{g^F_{pe,j,t}}{g^F_{te,j,t}} \right)^{\sigma} \left(LD^F_{pe,j,t} \right)$$

Replacing the above in (22) and defining $CT^F_{j,t} = \dfrac{\sum_l g^F_{l,j,t} LD^F_{l,j,t}}{LDC^F_{j,t}}$, we obtain

$$LD^F_{l,j,t} = \left[\frac{\beta^{LD^F}_{l,j} CT^F_{j,t}}{g^F_{l,j,t}} \right]^{\sigma^{LD^F}_j} \left[B^{LDF}_j A^F_l \varepsilon^F_l f^F_{l,j,t} \right]^{\sigma^{LD^F}_j - 1} LDC^F_{j,t}$$

where

$$CT^F_{j,t} = \frac{1}{B^{LDF}_j} \sum_l \left[\left(\beta^{LD^F}_{l,j} \right)^{\sigma^{LD^F}_j} \left(A_l \varepsilon_{Fl} f_{Fl,t} \right)^{\left(\sigma^{LD^F}_j - 1 \right)} \left(g^F_{l,j,t} \right)^{1-\sigma^{LD^F}_j} \right]^{\frac{1}{1-\sigma^{LD^F}_j}}$$

Now we find the wages. Replacing (20) in (21), we find

$$\frac{\partial f^F_{l,j,t}}{\partial W^F_{l,j,t}} \frac{W^F_{l,j,t}}{f^F_{l,j,t}} = \frac{\partial g^F_{l,j,t}}{\partial W^F_{l,j,t}} \frac{W^F_{l,j,t}}{g^F_{l,j,t}}. \tag{23}$$

The equilibrium efficiency wage is found at the point where the elasticity of the rate of effort with respect to the wage equals the elasticity of unit labor costs with respect to the wage. To find the equilibrium wage explicitly, (9) and $g^F_{l,j,t}$ are used.

$$\frac{\partial g^F_{l,j,t}}{\partial W^F_{l,j,t}} \frac{W^F_{l,j,t}}{g^F_{l,j,t}} = \left[1 - \pi + \pi f^F_{l,j,t} + W^F_{l,j,t} \pi \frac{\partial f^F_{l,j,t}}{\partial W^F_{l,j,t}} \right] \left(1 + ttiw_{l,j,t} \right) \frac{W^F_{l,j,t}}{g^F_{l,j,t}} \tag{24}$$

Then, we replace (9) and (24) in (23) to obtain

$$\frac{1}{\left(1 + ttiw_{l,j,t} \right)} \frac{g^F_{l,j,t}}{W^F_{l,j,t}} = \frac{1}{\pi \left(1 - f^F_{l,j,t} \right)} \left[1 - \pi + \pi f^F_{l,j,t} + \pi \left(1 - f^F_{l,j,t} \right) \pi f^F_{l,j,t} \right]$$

Now, replacing (19) in the above equation we arrive at

$$\pi \left(1 - f^F_{l,j,t} \right) = \frac{1}{2 - \pi + \frac{c^F_l \pi \left(1 - f^F_{l,j,t} \right) + E^F_{l,t}}{\left(1 + ttiw_{l,j,t} \right) W^F_{l,j,t}}}$$

Finally, this equation and (8) imply

$$\frac{\pi}{\left(1 - \varepsilon^F_l \right) \left(\frac{W^F_{l,j,t}}{W^I_{l,t}} \right)^{\pi} + 1} = \frac{1}{2 - \pi + \frac{c^F_l \pi \left(1 - f^F_{l,j,t} \right) + E_{l,t}}{\left(1 + ttiw^F_{l,j,t} \right) W^F_{l,j,t}}} \tag{25}$$

The left side of this equation is a decreasing function of $W^F_{l,j,t}$ and the right side is an increasing function of $W^F_{l,j,t}$.

6 The microsimulation model Background

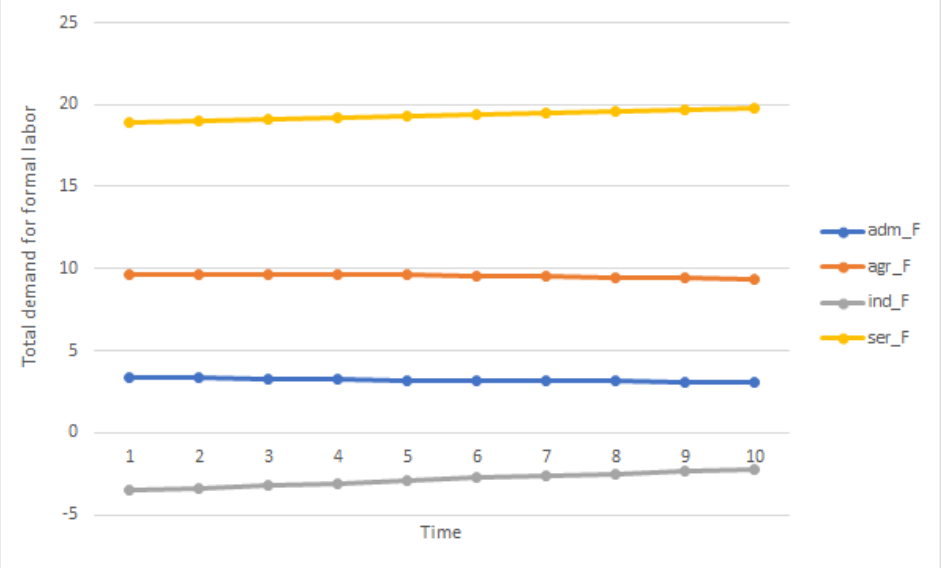
The results of the macro model will feed into the microsimulation model. We will use a methodology similar to Tiberti, Cicowiez, and Cockburn ((Tiberti, Cicowiez, Cockburn et al. 2018) 2018) but emphasizing the informal sector, formal temporary jobs, and formal permanent jobs. Incomes by type of worker are going to be estimated, changes in price vectors from the CGE are going to introduced into the model, and distributional analysis will be performed.

7 Preliminary results

In this section, some preliminary results (using fictitious data) will be discussed regarding the effects of modification of dismissal costs on some results of the labor market in the presence of informality and temporary work. Specifically, the consequences of eliminating the cost of firing permanent workers on labor demands, wages, unemployment rate, and households and firms' income will be analyzed. Initially, it is assumed that companies do not have to pay any cost for firing temporary workers. This exercise is interesting because one of the main arguments in favor of reducing firing costs is that they discourage hiring formal labor and make firms unable to adequately adapt to the economic cycle when needing to fire workers. However, in the presence of informality and temporary work, eliminating firing costs can cause losses in company income, discourage the supply of formal labor, and negatively affect income distribution among households.

In this respect, Figure 5 shows the evolution of formal labor demand after eliminating firing costs for formal sectors. It can be seen that the elimination of dismissal costs has a positive effect on formal labor demand in the services (ser_F), agriculture (agr_f), and public administration (adm_F) sectors. However, the industry sector (ind_F) shows negative variations in formal wages. Most sectors' behavior is in line with the expected since formal labor becomes less expensive.

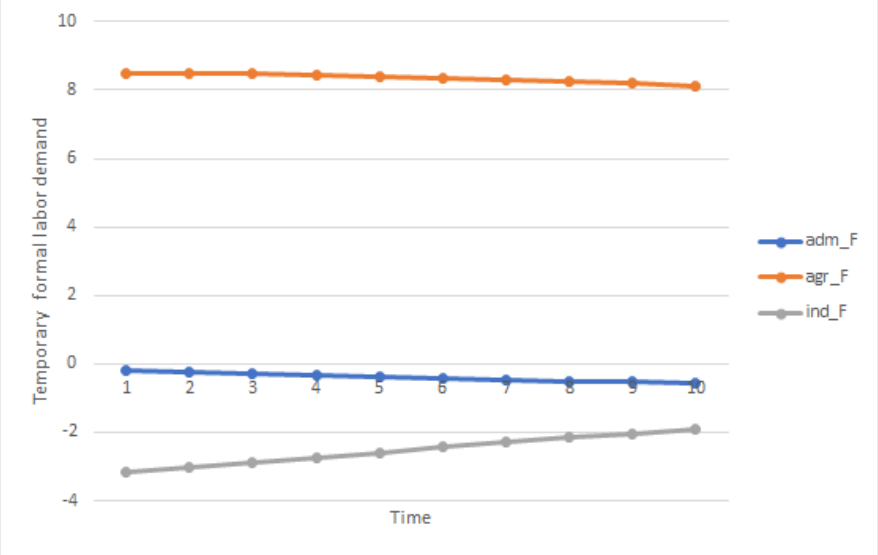
Figure 5: Total demand for formal labor



Source: Own elaboration

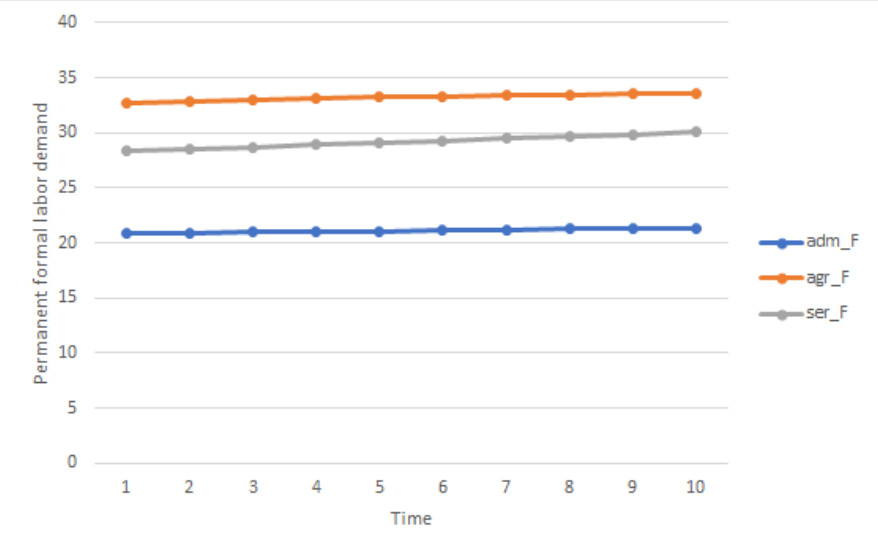
Moreover, the demand for temporary work decreases in the public administration and industry sectors but increases in the agriculture sector (Figure 6). The decrease in demand for temporary work in the public administration and industry sectors occurs because permanent work is more productive, which is why companies will prefer this type of work. This behavior is as expected given the literature review. The agricultural sector behaves especially because, given the initial data of the model, it is intensive in temporary work. However, the demand for permanent work (Figure 7) increases in the public administration, agriculture, and services sectors. It is observed that the increase in the demand for permanent work in the agricultural sector is much more significant than the increase in the demand for temporary work in this sector. These results are as expected because, although all formal work becomes less expensive, permanent work is more productive and therefore will be demanded to a greater extent by companies.

Figure 6: Temporary formal labor demand

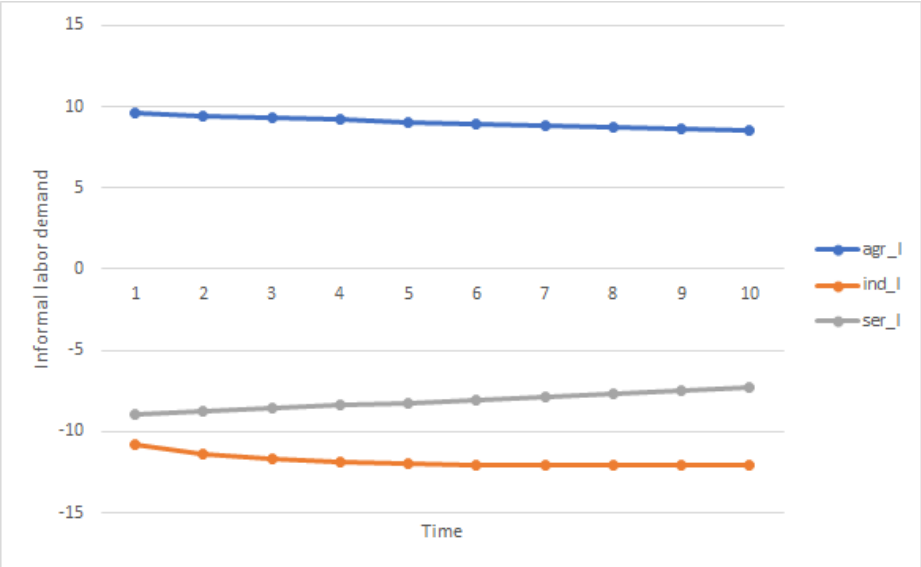


Source: Own elaboration

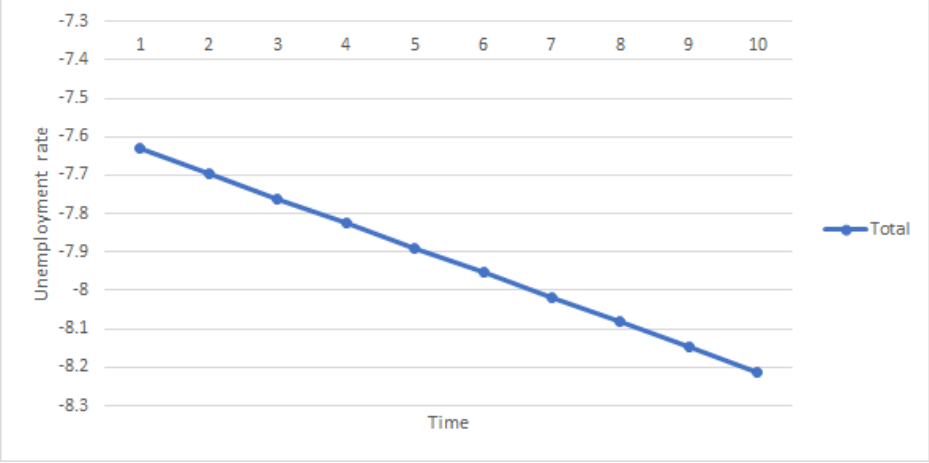
Figure 7: Permanent formal labor demand



Source: Own elaboration
Figure 8: Informal labor demand



Source: Own elaboration
Figure 9: Unemployment rate



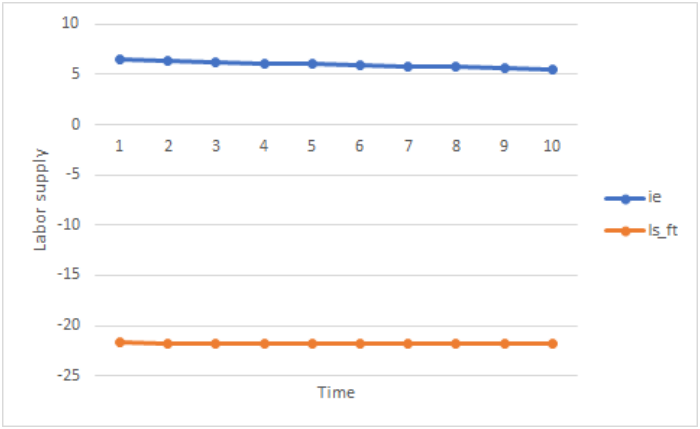
Source: Own elaboration

In addition, as can be seen in figure 8, the demand for informal work decreases in the informal sectors of services and industry but increases in the informal agricultural sector. The effect on the service and industrial sectors is as expected due to the corresponding rise in the demand for formal work. In

general terms, taking into account the graphs that have been analyzed in this section, it can be said that formal work becomes cheaper in general terms and increases its demand in most formal industries. In contrast, informal work is demanded to a lesser extent in most informal industries. It follows that eliminating permanent job layoff costs can have positive results in terms of increasing formal job recruitment. Added to this is the decrease in the unemployment rate (Figure 9).

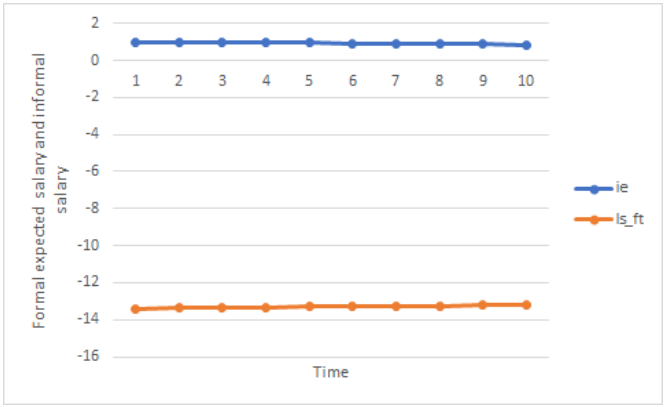
Up to this point, the results in terms of the labor market have been positive: less demand for informal labor and more demand for formal labor in most sectors. However, it is still necessary to look at the consequences related to the labor supply, wages, and income of households and firms. Figure 10 shows that the informal labor supply increases whereas the formal labor supply decreases dramatically. This occurs because eliminating dismissal costs for permanent workers makes the formal labor market less attractive, and households prefer to work in the informal sector. Likewise, Figure 11 informs that the formal expected wage experiences a drastic decrease, whereas the informal wage shows an increase, which justifies what is expressed in Figure 10.

Figure 10: Formal labor supply



Source: Own elaboration

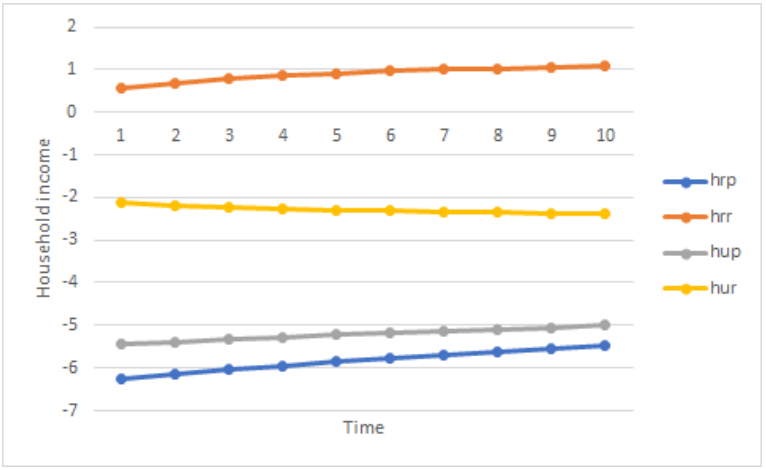
Figure 11: Formal expected salary and informal salary



Source: Own elaboration

Another important aspect is the one referring to the income of households and firms. From Figure 12, it can be deduced that the poor rural households (hrp) and the poor urban households (hup) are those that suffer a more vehement reduction in their income, whereas that the rich urban households (hur) present a decrease in their income but to a lesser extent. Finally, the Rich rural households (hrr) experience an increase in their income. Therefore, the less favored households suffer most vehemently from the adverse effects of the reform.

Figure 12: Household income

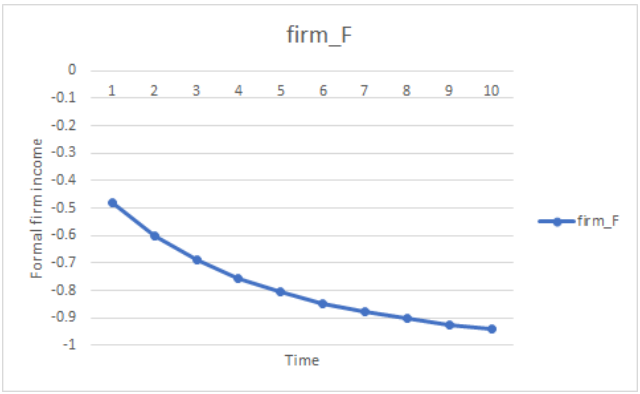


Source: Own elaboration

Finally, the income of formal firms is negatively affected, which reflects that the decrease in their costs is not necessarily reflected in an increase in income

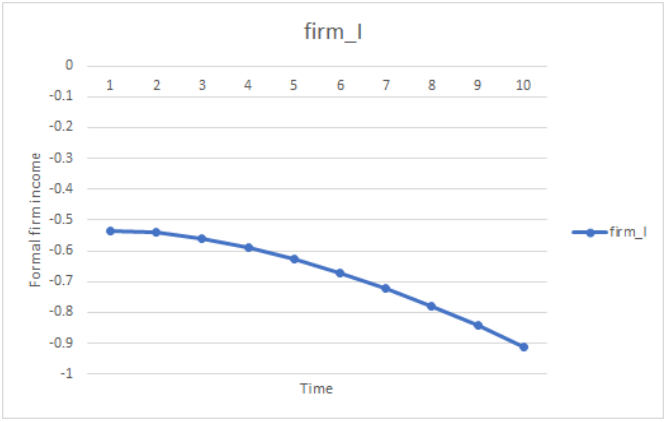
(Figure 13). Moreover, the same pattern is shown in Figure 14, that is, informal firms experience a negative variation in their income. Roughly, the incomes of most households are negatively affected, especially those of less favored households, and firms' incomes do not experience any improvement, rather they worsen. Therefore, although some positive results are obtained in the labor market, after the reform, people offer less work in the formal sector, and the income of households and firms falls.

Figure 13: Formal firm income



Source: Own elaboration

Figure 14: Informal firm income



Source: Own elaboration

8 Concluding remarks

One of the most significant constraints in the fight against poverty is the lack of an equitable and inclusive labor market. The persistence of high levels of informality means that many of the population in developing countries have poor working conditions and low incomes. This is compounded by the growing expansion of temporary contracts in these countries. This form of contracting is associated with wage penalties, few training opportunities, job instability, and health problems. Therefore, formalization alone is not enough to reduce the gaps. There is a need for a joint policy that takes these two spheres into account.

The simulation exercise with fictitious data allows analyzing some important relationships that can be useful when executing labor reforms. In fact, although it may be evident that eliminating the costs of firing permanent workers motivates companies to hire more formal labor, the results show that this can make the formal market less attractive to households, cause lower wages in the economy and lower the income of the poorest. In this sense, although the unemployment rate decreases, this occurs at the expense of creating precarious jobs.

References

Agénor, P.-R., Izquierdo, A. & Jensen, H. T. (2009), *Adjustment policies, poverty, and unemployment: the IMMPA framework*, John Wiley & Sons.

Akbulut, H. & Eğen, H. T. (2021), 'Import tariffs and informal labour market: A computable general equilibrium (cge) analysis for turkey', *Review of Economic Analysis* **13**(1).

Arguello Cuervo, L. R., Jimenez, D., Torres, E. & Gasca, M. (2016), 'Dutch disease, informality, and employment intensity in colombia', *Dutch Disease Informality and Employment Intensity in Colombia (June, 2016); 33 pp.* .

Atuesta, L. & Hewings, G. J. (2013), 'Economic welfare analysis of the legalization of drugs: A cge microsimulation model for colombia', *Economic Systems Research* **25**(2), 190–211.
URL: *https://doi.org/10.1080/09535314.2012.728130*

Bacchetta, M., Ernst, E., Bustamante, J. P. et al. (2009), *Globalization and informal jobs in developing countries*, International Labour Organization Geneva.

Blanchard, O. & Landier, A. (2002), 'The perverse effects of partial labour market reform: Fixed-term contracts in france', *The Economic Journal*

112(480), F214–F244.
URL: *http://www.jstor.org/stable/798373*

Boeri, T. & Garibaldi, P. (2007), 'Two tier reforms of employment protection: a honeymoon effect?*', *The Economic Journal* **117**(521), F357–F385.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1468-0297.2007.02060.x*

Bonnet, F., Vanek, J. & Chen, M. (2019), 'Women and men in the informal economy: A statistical brief', *WIEGO and ILO* .

Booth, A. L., Francesconi, M. & Frank, J. (2002), 'Temporary jobs: stepping stones or dead ends?', *The economic journal* **112**(480), F189–F213.

Bosch, M. & Esteban-Pretel, J. (2012), 'Job creation and job destruction in the presence of informal markets', *Journal of Development Economics* **98**(2), 270–286.
URL: *https://www.sciencedirect.com/science/article/pii/S030438781100085X*

Bourguignon, F. & Savard, L. (2008), 'Distributional effects of trade reform: An integrated macro–micro model applied to the philippines', *The Impact of Macroeconomic Policies on Poverty and Income Distribution: Macro-micro Evaluation Techniques and Tools* pp. 177–212.

Bussolo, M., Lay, J. & Van der Mensbrugghe, D. (2006), *Structural change and poverty reduction in Brazil: the impact of the Doha Round*, Vol. 3833, World Bank Publications.

Bussolo, M. & Round, J. I. (2006), *Globalisation and Poverty*, Routledge.

Cahuc, P., Charlot, O. & Malherbet, F. (2016), 'Explaining the spread of temporary jobs and its impact on labor turnover', *International Economic Review* **57**(2), 533–572.

Cahuc, P. & Postel-Vinay, F. (2002), 'Temporary jobs, employment protection and labor market performance', *Labour Economics* **9**(1), 63–91.
URL: *https://www.sciencedirect.com/science/article/pii/S0927537101000513*

Cappellari, L., DellAringa, C. & Leonardi, M. (2012), 'Temporary employment, job flows and productivity: A tale of two reforms*', *The Economic Journal* **122**(562), F188–F215.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1468-0297.2012.02535.x*

Castellani, F., Lotti, G. & Obando, N. (2020), 'Fixed or open-ended? labor contract and productivity in the colombian manufacturing sector', *Journal of Applied Economics* **23**(1), 199–223.
URL: *https://doi.org/10.1080/15140326.2020.1729582*

Cogneau, D., Robilliard, A.-S. et al. (2004), 'Poverty alleviation policies in madagascar: a micro-macro simulation model', *The Impact of Macroeconomic Policies on Poverty and Income Distribution: Macro-micro Evaluation Techniques and Tools* .

Decaluwé, B., Lemelin, A., Robichaud, V. & Maisonnave, H. (2013), 'The pep standard single-country, recursive dynamic cge model', *Jul-2013.[Online]. Available: http://www. pepnet. org/pep-1-t-single-country-recursive-dynamic-version.* .

Dolado, J. J., GarcÃıa-Serrano, C. & Jimeno, J. F. (2002), 'Drawing lessons from the boom of temporary jobs in spain', *The Economic Journal* **112**(480), F270–F295.
URL: *http://www.jstor.org/stable/798375*

El Badaoui, E. & Magnani, R. (2019), 'Tax policies and informality in south africa', *Journal of International Development* **0**(0).
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1002/jid.3416*

Engellandt, A. & Riphahn, R. T. (2005), 'Temporary contracts and employee effort', *Labour Economics* **12**(3), 281–299.
URL: *https://www.sciencedirect.com/science/article/pii/S092753710400003X*

Eslava, M., Haltiwanger, J., Kugler, A. & Kugler, M. (2014), 'The effects of regulations and business cycles on temporary contracts, the organization of firms and productivity'.
URL: *http://hdl.handle.net/10419/127670*

FEDESARROLLO (2018), Informe mensual del mercado laboral: productividad y salario mÃınimo, Technical report, FEDESARROLLO.
URL: *https://www.repository.fedesarrollo.org.co/handle/11445/3527?show=full*

Giraldo, M. C. & García, J. A. B. (2018), 'Fiscal policy and inequality in a cge model for colombia'.

Hernández Díaz, G. A. (2011), 'Impuestos parafiscales y mercado laboral: Un análisis de equilibrio general computable'.

Hopenhayn, H. & Rogerson, R. (1993), 'Job turnover and policy evaluation: A general equilibrium analysis', *Journal of Political Economy* **101**(5), 915–938.
URL: *http://www.jstor.org/stable/2138602*

ILO (2016), 'Non-standard employment around the world: understanding challenges, shaping prospects'.

ILO (2018*a*), 'Informality and non-standard forms of employment'.

ILO (2018*b*), 'Women and men in the informal economy: A statistical picture'.

Kleinknecht, A., Oostendorp, R. M., Pradhan, M. P. & Naastepad, C. (2006), 'Flexible labour, firm performance and the dutch job creation miracle', *International Review of Applied Economics* **20**(2), 171–187.
URL: *https://doi.org/10.1080/02692170600581102*

Kugler, A. D. (2007), *3. The Effect of Job Security Regulations on Labor Market Flexibility: Evidence from the Colombian Labor Market Reform*, University of Chicago Press.

Maurizio, R. (2016), *Non-standard forms of employment in Latin America: Prevalence, characteristics and impacts on wages*, International Labour Organization, Argentina.

Maurizio, R. d. L. (2019), 'Temporary employment and its impact on wages in latin america', *Iberoamerican Journal of Development Studies* .

Meghir, C., Narita, R. & Robin, J.-M. (2015), 'Wages and informality in developing countries', *American Economic Review* **105**(4), 1509–46.
URL: *http://www.aeaweb.org/articles?id=10.1257/aer.20121110*

OECD (2016), *OECD Reviews of Labour Market and Social Policies: Colombia 2016*, OECD Publishing, Paris.
URL: *https://www.oecd-ilibrary.org/content/publication/9789264244825-en*

OECD & ILO (2019), *Tackling Vulnerability in the Informal Economy"*, publisher.
URL: *https://www.oecd-ilibrary.org/content/publication/939b7bcd-en*

Osorio-Copete, L. M. (2016), 'Reforma tributaria e informalidad laboral en colombia: Un análisis de equilibrio general dinámico y estocástico', *Ensayos sobre política Económica* **34**, 126 – 145.

Paquet, M. & Savard, L. (2009), 'Impact of informal re-exports between benin and nigeria: A cge analysis', *Cahier de recherche/Working Paper* **9**, 14.

Rodríguez, E. & López, L. (2016), 'El empleo temporal en la industria colombiana, 2000-2013', *Semestre Económico* **19**(39), 31–60.

Rodríguez Guerrero, D. A. (2014), 'Ley 1607 de 2012 de reforma tributaria en colombia: un análisis de equilibrio general computable'.

Stifel, D. C. & Thorbecke, E. (2003), 'A dual-dual cge model of an archetype african economy: trade reform, migration and poverty', *Journal of Policy Modeling* **25**(3), 207–235.
URL: *https://www.sciencedirect.com/science/article/pii/S0161893803000140*

Tiberti, L., Cicowiez, M., Cockburn, J. et al. (2018), 'A top-down with behaviour (tdb) microsimulation toolkit for distributive analysis', *International Journal of Microsimulation* **11**(2), 191–213.

Ulyssea, G. (2018), 'Firms, informality, and development: Theory and evidence from brazil', *American Economic Review* **108**(8), 2015–47.
URL: *http://www.aeaweb.org/articles?id=10.1257/aer.20141745*

9 Appendix 1: Equations

9.1 Production

$$VA^F_{j,t} = v^F_j XST^F_{j,t} \tag{26}$$

$$VA^I_{j,t} = v^I_j XST^I_{j,t} \tag{27}$$

$$CI^F_{j,t} = io^F_j XST^F_{j,t} \tag{28}$$

$$CI^I_{j,t} = io^I_j XST^I_{j,t} \tag{29}$$

$$VA^F_{j,t} = B^{VA^F}_j \left[\beta^{VA^F}_j \left(LDC^F_{j,t}\right)^{-\rho^{VA^F}_j} + \left(1-\beta^{VA^F}_j\right)\left(KDC^F_{j,t}\right)^{-\rho^{VA^F}_j}\right]^{-\frac{1}{\rho^{VA^F}_j}} \tag{30}$$

$$VA^I_{j,t} = B^{VA^I}_j \left[\beta^{VA^I}_j \left(LDC^I_{j,t}\right)^{-\rho^{VA^I}_j} + \left(1-\beta^{VA}_j\right)\left(KDC^I_{j,t}\right)^{-\rho^{VA^I}_j}\right]^{-\frac{1}{\rho^{VA^I}_j}} \tag{31}$$

$$LDC^F_{j,t} = \left[\frac{\beta^{VA^F}_j}{1-\beta^{VA^F}_j}\frac{RC^F_{j,t}}{CT_{j,t}}\right]^{\sigma^{VA^F}_j} KDC^F_{j,t} \tag{32}$$

$$LDC^I_{j,t} = \left[\frac{\beta^{VA^I}_j}{1-\beta^{VA^I}_j}\frac{RC^I_{j,t}}{WC^I_{j,t}}\right]^{\sigma^{VA^I}_j} KDC^I_{j,t} \tag{33}$$

$$LDC^F_{j,t} = B^{LDF}_j \sum_l \left(\beta^{LD^F}_{l,j}\left(A^F_l \varepsilon^F_l f^F_{l,j,t} LD^F_{l,j,t}\right)^{-\rho^{LD^F}_j}\right)^{-\frac{1}{\rho^{LD^F}_j}} \tag{34}$$

$$LDC^I_{j,t} = LD^I_{l,j,t} \tag{35}$$

$$LD^F_{l,j,t} = \left[\frac{\beta^{LD^F}_{l,j} CT^F_{j,t}}{g^F_{l,j,t}}\right]^{\sigma^{LD^F}_j} \left[B^{LDF}_j A^F_l \varepsilon^F_l f^F_{l,j,t}\right]^{\sigma^{LD^F}_j - 1} LDC^F_{j,t} \tag{36}$$

$l = te, pe, ie$

$$f^F_{l,j,t} = \frac{1}{1 + \frac{1}{\left(1-\varepsilon^F_l\right)} \frac{\left(W^I_{l,t}\right)^{\pi}}{\left(W^F_{l,j,t}\right)^{\pi}}} \tag{37}$$

$$g^F_{l,j,t} = \left[1 - \pi + \pi f^F_{l,j,t}\right] WTI^F_{l,j,t} + c^F_l \pi \left(1 - f^F_{l,j,t}\right) + E^F_{l,t} \tag{38}$$

$$CT^F_{j,t} = \frac{1}{B^{LDF}_j} \sum_l \left[\left(\beta^{LD^F}_{l,j}\right)^{\sigma^{LD^F}_j} \left(A_l \varepsilon_{Fl} f_{Fl,t}\right)^{\left(\sigma^{LD^F}_j - 1\right)} \left(g^F_{l,j,t}\right)^{1-\sigma^{LD^F}_j} \right]^{\frac{1}{1-\sigma^{LD^F}_j}} \tag{39}$$

$$\frac{\pi}{\left(1-\varepsilon^F_l\right)\left(\frac{W^F_{l,j,t}}{W^I_{l,t}}\right)^{\pi} + 1} = \frac{1}{2 - \pi + \frac{c^F_l \pi \left(1 - f^F_{l,j,t}\right) + E_{l,t}}{\left(1 + ttiw^F_{l,j,t}\right) W^F_{l,j,t}}} \tag{40}$$

$$KDC^F_{j,t} = B^{KD^F}_j \left[\sum_k \beta^{KD^F}_{k,j} KD^{-\rho^{KD^F}_j}_{k,j,t} \right]^{-\frac{1}{\rho^{KD^F}_j}} \tag{41}$$

$$KD^F_{k,j,t} = \left[\frac{\beta^{KD^F}_{k,j} RC^F_{j,t}}{RTI^F_{k,j,t}} \right]^{\sigma^{KD^F}_j} \left(B^{KD^F}_j\right)^{\sigma^{KD^F}_j - 1} KDC^F_{j,t} \tag{42}$$

$$KDC^I_{j,t} = B^{KD^I}_j \left[\sum_k \beta^{KD^I}_{k,j} \left(KD^I_{k,j,t}\right)^{-\rho^{KD^I}_j} \right]^{-\frac{1}{\rho^{KD^I}_j}} \tag{43}$$

$$KD^I_{k,j,t} = \left[\frac{\beta^{KD^I}_{k,j} RC^I_{j,t}}{R^I_{k,j,t}} \right]^{\sigma^{KD^I}_j} \left(B^{KD^I}_j\right)^{\sigma^{KD^I}_j - 1} KDC^I_{j,t} \tag{44}$$

$$DI^F_{i,j,t} = aij^F_{i,j} CI^F_{j,t} \tag{45}$$

$$DI^I_{i,j,t} = aij^I_{i,j} CI^I_{j,t} \tag{46}$$

9.2 Labor supply

$$\Phi^F_{j,t} = \frac{LD^F_{te,j,t}}{LDC^F_{j,t}} \tag{47}$$

$$\Omega^F_{j,t} = \frac{LDC^F_{j,t}}{\sum_j LDC^F_{j,t}} \tag{48}$$

$$\mu_t = LS^F_{ls\_ft,t} - \left(\sum_j LDC^F_{j,t} \right) \tag{49}$$

$$PU_t^F = \frac{\mu_t}{LS_{ls\_ft,t}^F} = \frac{LS_{ls\_ft,t}^F - \left(\sum_j LDC_{j,t}^F\right)}{LS_{ls\_ft,t}^F} \tag{50}$$

$$W_t^{e^F} = \left(PU_t^F \times 0\right) + \left(1 - PU_t^F\right) \sum_j \Omega_{j,t}^F \left[\Phi_{j,t}^F \left(\left(\pi f_{te,j,t}^F + 1 - \pi\right) W_{te,j,t}^F + c_{te}^F \pi \left(1 - f_{te,j,t}^F\right)\right) + \left(1 - \Phi_{j,t}^F\right) \left(\left(\pi f_{pe,j,t}^F + 1 - \pi\right) W_{pe,j,t}^F + c_{pe}^F \pi \left(1 - f_{pe,j,t}^F\right)\right)\right] \tag{51}$$

$$\widehat{W}_t = \left(\psi_I \left(\frac{W_{l,t}^I}{\psi_I}\right)^{1+\sigma^{LS}} + \psi_F \left(\frac{W_t^{e^F}}{\psi_F}\right)^{1+\sigma^{LS}}\right)^{\frac{1}{1+\sigma^{LS}}} \tag{52}$$

$$\widehat{W}_t LS_{h,t} = W_{l,t}^I LS_{h,I,t} + W_t^{e^F} LS_{h,F,t} \tag{53}$$

$$LS_{h,F,t} = \left(\frac{1}{\psi_F} \frac{W_t^{e^F}}{\widehat{W}}\right)^{\sigma^{LS}} LS_{h,t} \tag{54}$$

$$LS_{h,I,t} = \left(\frac{1}{\psi_I} \frac{W_{l,t}^I}{\widehat{W}}\right)^{\sigma^{LS}} LS_{h,t} \tag{55}$$

$$LS_{h,t} = \left(\psi_I \left(LS_{h,I,t}\right)^{\frac{1+\sigma^{LS}}{\sigma^{LS}}} + \psi_F \left(LS_{h,F,t}\right)^{\frac{1+\sigma^{LS}}{\sigma^{LS}}}\right)^{\frac{\sigma^{LS}}{1+\sigma^{LS}}} \tag{56}$$

$$LS_{ls\_ft,t}^F = \sum_h LS_{h,F,t} \tag{57}$$

$$LS_{I,t} = \sum_h LS_{h,I,t} \tag{58}$$

$$LS_t = LS_{F,t} + LS_{I,t} \tag{59}$$

9.3 Income and Savings

9.3.1 Households

$$YH_{h,t} = YHL_{h,t} + YHK_{h,t} + YHTR_{h,t} \tag{60}$$

$$YHL_{h,t} = \lambda_{h,l}^{WL^I} \left(W_{l,t}^I LS_{I,t}\right) + \lambda_{h,pe}^{WL^F} \left(W_t^{e^F} LS_{ls\_ft,t}^F\right) + \lambda_{h,te}^{WL^F} \left(W_t^{e^F} LS_{ls\_ft,t}^F\right) \tag{61}$$

$$YHL_{h,t} = \sum_l \lambda_{h,l}^{WL} \left(W_{l,t} \sum_j LD_{l,j,t}\right)$$

$$YHK_{h,t} = YHK^F_{h,t} + YHK^I_{h,t} \tag{62}$$

$$YHK^F_{h,t} = \sum_k \lambda^{RK^F}_{h,k} \left(\sum_j R^F_{k,j,t} KD^F_{k,j,t} \right) \tag{63}$$

$$YHK^I_{h,t} = \sum_k \lambda^{RK^I}_{h,k} \left(\sum_j R^I_{k,j,t} KD^I_{k,j,t} \right) \tag{64}$$

$$YHTR_{h,t} = YHTR^I_{h,t} + YHTR^F_{h,t} \tag{65}$$

$$YHTR^I_{h,t} = \sum_{ag} TR^I_{h,ag,t} = 0 \tag{66}$$

$$YHTR^F_{h,t} = \sum_{ag} TR^F_{h,ag,t} \tag{67}$$

$$YDH_{h,t} = YH_{h,t} - TDH_{h,t} - TR_{gvt,h,t} \tag{68}$$

$$CTH_{h,t} = YDH_{h,t} - SH_{h,t} - \sum_{agng} TR_{agng,h,t} \tag{69}$$

$$SH_{h,t} = PIXCON^{\eta}_t sh0_{h,t} + sh1_{h,t} YDH_{h,t} \tag{70}$$

9.3.2 Firms

$$YF^F_{FF,t} = YFK^F_{FF,t} + YFTR^F_{FF,t} \tag{71}$$

$$YFK^F_{FF,t} = \sum_k \lambda^{RK^F}_{FF,k} \left(\sum_j R^F_{k,j,t} KD^F_{k,j,t} \right) \tag{72}$$

$$YFTR^F_{FF,t} = \sum_{ag} TR_{FF,ag,t} \tag{73}$$

$$YDF^F_{FF,t} = YF^F_{FF,t} - TDF_{FF,t} \tag{74}$$

$$SF^F_{FF,t} = YDF^F_{FF,t} - \sum_{ag} TR_{ag,FF,t} \tag{75}$$

$$YF^I_{FI,t} = YFK^I_{FI,t} + YFTR^I_{FI,t} \tag{76}$$

$$YFK^I_{FI,t} = \sum_k \lambda^{RK^I}_{FI,k} \left(\sum_j R^I_{k,j,t} KD^I_{k,j,t} \right) \tag{77}$$

$$YFTR^{I}_{FI,t} = \sum_{ag} TR_{FI,ag,t} = 0 \tag{78}$$

$$YDF^{I}_{FI,t} = YF^{I}_{FI,t} \tag{79}$$

$$SF_{FI,t} = YDF^{F}_{FI,t} - \sum_{ag} TR_{ag,FI,t} \tag{80}$$

9.3.3 Government

$$YG_t = YGK_t + TDHT_t + TDFT_t + TPRODN_t + TPRCTS_t + YGTR_t \tag{81}$$

$$YGK_t = \sum_{k} \lambda^{RK^F}_{gvt,k} \left(\sum_{j} R^{F}_{k,j,t} KD^{F}_{k,j,t} \right) \tag{82}$$

$$TDHT_t = \sum_{h} TDH_{h,t} \tag{83}$$

$$TDFT_t = \sum_{FF} TDF_{FF,t} \tag{84}$$

$$TPRODN_t = TIWT_t + TIKT_t + TIPT_t \tag{85}$$

$$TIWT_t = \sum_{l,j} TIW^{F}_{l,j,t} \tag{86}$$

$$TIKT_t = \sum_{k,j} TIK^{F}_{k,j,t} \tag{87}$$

$$TIPT_t = \sum_{j} TIP^{F}_{j,t} \tag{88}$$

$$TPRCTS_t = TICT_t + TIMT_t + TIXT_t \tag{89}$$

$$TICT_t = \sum_{i} TIC_{i,t} \tag{90}$$

$$TIMT_t = \sum_{i} TIM_{i,t} \tag{91}$$

$$TIXT_t = \sum_{i} TIX_{i,t} \tag{92}$$

$$YGTR_t = \sum_{agng} TR_{gvt,\ agng\ ,t} \tag{93}$$

$$TDH_{h,t} = PIXCON_t^{\eta} ttdh0_{h,t} + ttdh1_{h,t} \left[YH_{h,t}^F \right] \tag{94}$$

$$YH_{h,t}^F = \lambda_{h,pe}^{WL^F} W_t^{e^F} LS_{h,t}^F + \lambda_{h,te}^{WL^F} W_t^{e^F} LS_{h,t}^F + YHK_{h,t}^F + YHTR_{h,t}^F \tag{95}$$

$$TDF_{FF,t} = PIXCON_t^{\eta} ttdf0_{FF,t} + ttdf1_{FF,t} YFK_{FF,t}^F \tag{96}$$

$$TIW_{l,j,t}^F = ttiw_{l,j,t}^F W_{l,j,t}^F \left[\left(1 - \pi + \pi f_{l,j,t}^F \right) LD_{l,j,t}^F \right] \tag{97}$$

$$TIK_{k,j,t}^F = \ ttik_{k,j,t}^F R_{k,j,t}^F KD_{k,j,t}^F \tag{98}$$

$$TIP_{j,t}^F = \ ttip_{j,t}^F PP_{j,t}^F XST_{j,t}^F \tag{99}$$

$$TIC_{i,t} = ttic_{i,t} \left[\begin{array}{l} \left(PL_{i,t}^F + \sum_{ij} PC_{ij,t} tmrg_{ij,i}^F \right) DD_{i,t}^F \\ + \left(\left(1 + ttim_{i,t} \right) PWM_{i,t} e_t + \sum_{ij} PC_{ij,t} tmrg_{ij,i}^F \right) IM_{i,t}^F \end{array} \right] \tag{100}$$

$$TIM_{\ i,t} = ttim_{i,t} PWM_{i,t} e_t IM_{i,t}^F \tag{101}$$

$$TIX_{i,t} = tix_{i,t} \left(PE_{i,t}^F + \sum_{ij} PC_{ij,t} tmrg_{ij,i}^{X^F} \right) EXD_{i,t}^F \tag{102}$$

$$SG_t = YG_t - \sum_{agng} TR_{agng,gvt,t} - G_t \tag{103}$$

9.3.4 Rest of the world

$$YROW_t = e_t \sum_i PWM_{i,t} IM_{i,t}^F + \sum_k \lambda_{row,k}^{RK^F} \left(\sum_j R_{k,j,t}^F KD_{k,j,t}^F \right) + \sum_{agd} TR_{row,agd,t} \tag{104}$$

$$SROW_t = YROW_t - \sum_i PE_{i,t}^{FOB} EXD_{i,t}^F - \sum_{agd} TR_{agd,row,t} \tag{105}$$

$$SROW_t = -CAB_t \tag{106}$$

9.3.5 Transfers

$$TR_{agng,h,t} = \lambda^{TR}_{agng,h} YDH_{h,t} \tag{107}$$

$$TR_{gvt,h,t} = PIXCON^{\eta}_{t} tr0_{h,t} + tr1_{h,t} YH_{h,t} \tag{108}$$

$$TR_{ag,FF,t} = \lambda^{TR}_{ag,FF} YDF^{F}_{FF,t} \tag{109}$$

$$TR_{ag,FI,t} = \lambda^{TR}_{ag,FI} YDF^{I}_{FI,t} \tag{110}$$

$$TR_{agng,gvt,t} = PIXCON\ {}^{\eta}_{t} TR^{O}_{agng,gvt}\ pop_t \tag{111}$$

$$TR_{agd,row,t}\ =\ PIXCON^{\eta}_{t}\ TR^{O}_{agd,row}\ pop_t \tag{112}$$

9.4 Demand

$$PC_{i,t} C_{i,h,t} = PC_{i,t} C^{MIN}_{i,h,t} + \gamma^{LES}_{i,h} \left(CTH_{h,t} - \sum_{ij} PC_{ij,t} C^{MIN}_{ij,h,t} \right) \tag{113}$$

$$GFCF_t = IT_t - \sum_{i} PC_{i,t} VSTK_{i,t} \tag{114}$$

$$PC_{i,t} INV^{PRI^F}_{i,t} = \gamma^{INVPRI^F}_{i} IT^{PRI^F}_{t} \tag{115}$$

$$PC_{i,t} INV^{PRI^I}_{i,t} = \gamma^{INVPRI^I}_{i} IT^{PRI^I}_{t} \tag{116}$$

$$PC_{i,t} INV^{PUB}_{i,t} = \gamma^{INVPUB}_{i} IT^{PUB}_{t} \tag{117}$$

$$INV_{i,t} = INV^{PRI^F}_{i,t} + INV^{PRI^I}_{i,t} + INV^{PUB}_{i,t} \tag{118}$$

$$PC_{i,t} CG_{i,t} = \gamma^{GVT}_{i} G_t \tag{119}$$

$$DIT_{i,t} = \sum_{j} DI^{F}_{i,j,t} + \sum_{j} DI^{I}_{i,j,t} \tag{120}$$

$$MRGN_{i,t} = \left[\sum_{ij} \mathrm{tmrg}^{F}_{i,ij}\ DD^{F}_{ij,t} + \sum_{ij} \mathrm{tmrg}^{I}_{i,ij}\ DD^{I}_{ij,t} \right] + \sum_{ij} \mathrm{tmrg}^{F}_{i,ij}\ IM^{F}_{ij,t} + \sum_{ij} tmrg^{X}_{i,ij} EXD^{F}_{ij,t} \tag{121}$$

9.5 Producer Supplies of Products and International Trade

$$XST^F_{j,t} = B^{XT^F}_j \left[\sum_i \beta^{XT^F}_{j,i} \left(XS^F_{j,i,t} \right)^{\rho^{XT^F}_j} \right]^{\frac{1}{\rho^{XT^F}_j}} \tag{122}$$

$$XST^I_{j,t} = B^{XT^I}_j \left[\sum_i \beta^{XT^I}_{j,i} \left(XS^I_{j,i,t} \right)^{\rho^{XT^I}_j} \right]^{\frac{1}{\rho^{XT^I}_j}} \tag{123}$$

$$XS^F_{j,i,t} = \frac{XST^F_{j,t}}{\left(B^{XT^F}_j \right)^{1+\sigma^{XT^F}_j}} \left[\frac{P^F_{j,i,t}}{\beta^{XT^F}_{j,i} PT^F_{j,t}} \right]^{\sigma^{XT^F}_j} \tag{124}$$

$$XS^I_{j,i,t} = \frac{XST^I_{j,t}}{\left(B^{XT^I}_j \right)^{1+\sigma^{XT^I}_j}} \left[\frac{P^I_{j,i,t}}{\beta^{XT^I}_{j,i} PT^I_{j,t}} \right]^{\sigma^{XT^I}_j} \tag{125}$$

$$XS^F_{j,i,t} = B^{X^F}_{j,i} \left[\beta^{X^F}_{j,i} \left(EX^F_{j,i,t} \right)^{\rho^{X^F}_{j,i}} + \left(1 - \beta^{X^F}_{j,i} \right) \left(DS^F_{j,i,t} \right)^{\rho^{X^F}_{j,x}} \right]^{\frac{1}{\rho^{X^F}_{j,i}}} \tag{126}$$

$$XS^I_{j,i,t} = DS^I_{j,i,t} \tag{127}$$

$$EX^F_{j,i,t} = \left[\frac{1 - \beta^{X^F}_{j,i}}{\beta^{X^F}_{j,i}} \frac{PE^F_{i,t}}{PL^F_{i,t}} \right]^{\sigma^{X^F}_{j,i}} DS^F_{j,i,t} \tag{128}$$

$$EXD^F_{i,t} = EXD^{FO^F}_i pop_t \left(\frac{e_t PWX_{i,t}}{PE^{FOB}_{i,t}} \right)^{\sigma^{XD^F}_i} \tag{129}$$

$$Q^F_{i,t} = B^{M^F}_i \left[\beta^{M^F}_i \left(IM^F_{i,t} \right)^{-\rho^{M^F}_i} + \left(1 - \beta^{M^F}_i \right) \left(DD^F_{i,t} \right)^{-\rho^{M^F}_i} \right]^{\frac{-1}{\rho^{M^F}_i}} \tag{130}$$

$$Q^I_{i,t} = DD^I_{i,t} \tag{131}$$

$$IM^F_{i,t} = \left[\frac{\beta^{M^F}_i}{1 - \beta^{M^F}_i} \frac{PD^F_{i,t}}{PM^F_{i,t}} \right]^{\sigma^{M^F}_i} DD^F_{i,t} \tag{132}$$

$$Q_{i,t} = B^Q_i \left[\beta^Q_i \left(Q^F_{i,t} \right)^{-\rho^Q_i} + \left(1 - \beta^Q_i \right) \left(DD^I_{i,t} \right)^{-\rho^Q_i} \right]^{\frac{-1}{\rho^Q_i}} \tag{133}$$

$$Q^F_{i,t} = \left[\frac{\beta^Q_i}{\left(1 - \beta^Q_i \right)} \frac{PD^I_{i,t}}{PQ^F_{i,t}} \right]^{\sigma^Q_i} DD^I_{i,t} \tag{134}$$

9.6 Prices

9.6.1 Production

$$PC_{i,t} = \frac{PQ^F_{i,t} Q^F_{i,t} + PD^I_{i,t} DD^I_{i,t}}{Q_{i,t}} \tag{135}$$

$$PP^F_{j,t} = \frac{PVA^F_{j,t} VA^F_{j,t} + PCI^F_{j,t} CI^F_{j,t}}{XST^F_{j,t}} \tag{136}$$

$$PP^I_{j,t} = \frac{PVA^I_{j,t} VA^I_{j,t} + PCI^I_{j,t} CI^I_{j,t}}{XST^I_{j,t}} \tag{137}$$

$$PT^F_{j,t} = \left(1 + \ ttip^F_{j,t}\right) PP^F_{j,t} \tag{138}$$

$$PT^I_{j,t} = PP^I_{j,t} \tag{139}$$

$$PCI^F_{j,t} = \frac{\sum_i PC_{i,t} DI^F_{i,j,t}}{CI^F_{j,t}} \tag{140}$$

$$PCI^I_{j,t} = \frac{\sum_i PC_{i,t} DI^I_{i,j,t}}{CI^I_{j,t}} \tag{141}$$

$$PVA^F_{j,t} = \frac{CT_{j,t} LDC^F_{j,t} + RC^F_{j,t} KDC^F_{j,t}}{VA^F_{j,t}} \tag{142}$$

$$PVA^I_{j,t} = \frac{W^I_{l,t} LDI_{I,j,t} + RC^I_{j,t} KDC^I_{j,t}}{VA^I_{j,t}} \tag{143}$$

$$WC^I_{j,t} = W^I_{l,t} \tag{144}$$

$$WTI^F_{l,j,t} = W^F_{l,j,t} \left(1 + ttiw^F_{l,j,t}\right) \tag{145}$$

$$RC^F_{j,t} = \frac{\sum_k RTI^F_{k,j,t} KD^F_{k,j,t}}{KDC^F_{j,t}} \tag{146}$$

$$RC^I_{j,t} = \frac{\sum_k R^I_{k,j,t} KD^I_{k,j,t}}{KDC^I_{j,t}} \tag{147}$$

$$RTI^F_{k,j,t} = R^F_{k,j,t} \left(1 + ttik^F_{k,j,t}\right) \tag{148}$$

$$RTI^I_{k,j,t} = R^I_{k,j,t} \tag{149}$$

9.6.2 International trade

$$PT^F_{j,t} = \frac{\sum_i P^F_{j,i,t} XS^F_{j,t}}{XST^F_{j,t}} \tag{150}$$

$$PT^I_{j,t} = \frac{\sum_i P^I_{j,i,t} XS^I_{j,t}}{XST^I_{j,t}} \tag{151}$$

$$P^F_{j,i,t} = \frac{PE^F_{i,t} EX^F_{j,i,t} + PL^F_{i,t} DS^F_{j,i,t}}{XS^F_{j,i,t}} \tag{152}$$

$$P^I_{j,i,t} = PL^I_{i,t} \tag{153}$$

$$PE^{FOB}_{i,t} = \left(PE^F_{i,t} + \sum_{ij} PC_{ij,t} tmrg^{X^F}_{ij,i} \right) (1 + ttix_{i,t}) \tag{154}$$

$$PD^F_{i,t} = (1 + ttic_{i,t}) \left(PL^F_{i,t} + \sum_{ij} PC_{ij,t} tmrg^F_{ij,i} \right) \tag{155}$$

$$PD^I_{i,t} = PL^I_{i,t} + \sum_{ij} PC_{ij,t} tmrg^I_{ij,i} = PQ^I_{i,t} \tag{156}$$

$$PM^F_{i,t} = (1 + ttic_{i,t}) \left((1 + ttim_{i,t})\, e_t PWM_{i,t} + \sum_{ij} PC_{ij,t} tmrg^F_{ij,i} \right) \tag{157}$$

$$PQ^F_{i,t} = \frac{PM^F_{i,t} IM^F_{i,t} + PD^F_{i,t} DD^F_{i,t}}{Q^F_{i,t}} \tag{158}$$

9.6.3 Price indexes

$$PIXGDP^F_t = \sqrt{\frac{\sum_j \left(PVA^F_{j,t} + \frac{TIP^F_{j,t}}{VA^F_{j,t}} \right) VA^{O^F}_j}{\sum_j \left(PVA^{O^F}_j VA^{O^F}_j + TIP^{O^F}_j \right)} \frac{\sum_j \left(PVA^F_{j,t} VA^F_{j,t} + TIP^F_{j,t} \right)}{\sum_j \left(PVA^{O^F}_j + \frac{TIP^{O^F}_j}{VA^{O^F}_j} \right) VA^F_{j,t}}} \tag{159}$$

$$PIXGDP^I_t = \sqrt{\frac{\sum_j \left(PVA^I_{j,t} \right) VA^{O^I}_j}{\sum_j \left(PVA^{O^I}_j VA^{O^I}_j \right)} \frac{\sum_j \left(PVA^I_{j,t} VA^I_{j,t} \right)}{\sum_j \left(PVA^{O^I}_j \right) VA^I_{j,t}}} \tag{160}$$

$$PIXCON_t = \frac{\sum_i PC_{i,t} \sum_h C^O_{i,h}}{\sum_{ij} PC^O_{ij} \sum_h C^O_{ij,h}} \tag{161}$$

$$PIXINV_t^{PRI^F} = \prod_i \left(\frac{PC_{i,t}}{PC^O_i} \right)^{\gamma_i^{INVPRI^F}} \tag{162}$$

$$PIXINV_t^{PRI^I} = \prod_i \left(\frac{PC_{i,t}}{PC^O_i} \right)^{\gamma_i^{INVPRI^I}} \tag{163}$$

$$PIXINV_t^{PUB} = \prod_i \left(\frac{PC_{i,t}}{PC^O_i} \right)^{\gamma_i^{INVPUB}} \tag{164}$$

$$PIXGVT_t = \prod_i \left(\frac{PC_{i,t}}{PC^O_i} \right)^{\gamma_i^{GVT}} \tag{165}$$

9.7 Equilibrium

$$Q_{i,t} = \sum_h C_{i,h,t} + CG_{i,t} + INV_{i,t} + VSTK_{i,t} + DIT_{i,t} + MRGN_{i,t} \tag{166}$$

$$LS_{I,t} = \sum_j LD_{I,j,t} \tag{167}$$

$$\sum_j KD^F_{k,j,t} = KS^F_{k,t} \tag{168}$$

$$\sum_j KD^I_{k,j,t} = KS^I_{k,t} \tag{169}$$

$$IT_t = \sum_h SH_{h,t} + (SF_{FI,t} + SF_{FF,t}) + SG_t + SROW_t \tag{170}$$

$$IT_t^{PRI^I} + IT_t^{PRI^F} = IT_t - IT_t^{PUB} - \sum_i PC_{i,t} VSTK_{i,t} \tag{171}$$

$$\sum_j DS^F_{j,i,t} = DD^F_{i,t} \tag{172}$$

$$\sum_j DS^I_{j,i,t} = DD^I_{i,t} \tag{173}$$

$$\sum_j EX^F_{j,i,t} = EXD^F_{i,t} \tag{174}$$

9.8 Gross domestic product

$$GDP_t^{BP^F} = \sum_j PVA_{j,t}^F VA_{j,t}^F + TIPT_t \tag{175}$$

$$GDP_t^{BP^I} = \sum_j PVA_{j,t}^I VA_{j,t}^I \tag{176}$$

$$GDP_t^{MP^F} = GDP_t^{BP^F} + TPRCTS_t \tag{177}$$

$$GDP_t^{IB^F} = \sum_{l,j} W_{l,,j,t}^F \left(1 - \pi + \pi f_{l,j,t}^F\right) LD_{l,j,t}^F + \sum_{k,j} R_{k,j,t}^F KD_{k,j,t}^F + TPRODN_t + TPRCTS_t \tag{178}$$

$$GDP_t^{FD^F} = \sum_i PC_{i,t} \left[\sum_h C_{i,h,t} + CG_{i,t} + \left(INV_{i,t}^{PRI^F} + INV_{i,t}^{PUB}\right) + VSTK_{i,t}\right] + \sum_i PE_{i,t}^{FOB} EXD_{i,t}^F - \sum_i e_t PWM_{i,t} IM_{i,t}^F \tag{179}$$

9.9 Real (Volume)Variables Computed from Price Indices

$$CTH_{h,t}^{REAL} = \frac{CTH_{h,t}}{PIXCON_t} \tag{180}$$

$$G_t^{REAL} = \frac{G_t}{PIXGVT_t} \tag{181}$$

$$GDP_t^{BP^F\_REAL} = \frac{GDP_t^{BP^F}}{PIXGDP_t^F} \tag{182}$$

$$GDP_t^{BP^I\_REAL} = \frac{GDP_t^{BP^I}}{PIXGDP_t^I} \tag{183}$$

$$GDP_t^{MP^F\_REAL} = \frac{GDP_t^{MP^F}}{PIXCON_t} \tag{184}$$

$$GFCF_t^{PRI^F\_REAL} = \frac{IT_t^{PRI^F}}{PIXINV_t^{PRI^F}} \tag{185}$$

$$GFCF_t^{PRI^I\_REAL} = \frac{IT_t^{PRI^I}}{PIXINV_t^{PRI^I}} \tag{186}$$

$$GFCF_t^{PUB\_REAL} = \frac{IT_t^{PUB}}{PIXINV_t^{PUB}} \tag{187}$$

9.10 Dynamic Equations

$$KD^F_{k,j,t+1} = KD^F_{k,j,t}\left(1-\delta^F_{k,j}\right) + IND^F_{k,j,t} \tag{188}$$

$$KD^I_{k,j,t+1} = KD^I_{k,j,t}\left(1-\delta^I_{k,j}\right) + IND^I_{k,j,t} \tag{189}$$

$$IT^{PUB}_t = PK^{PUB}_t \sum_{k,pub} IND_{k,pub,t} \tag{190}$$

$$IT^{PRI^F}_t = PK^{PRI^F}_t \sum_{k,bus} IND^F_{k,bus,t} \tag{191}$$

$$IT^{PRI^I}_t = PK^{PRI^I}_t \sum_{k,bus} IND^I_{k,bus,t} \tag{192}$$

$$PK^{PRI^F}_t = \frac{1}{A^{K\_PRI^F}} \prod_i \left[\frac{PC_{i,t}}{\gamma^{INVPRI^F}_i}\right]^{\gamma^{INVPRI^F}_i} \tag{193}$$

$$PK^{PRI^I}_t = \frac{1}{A^{K\_PRI^I}} \prod_i \left[\frac{PC_{i,t}}{\gamma^{INVPRI^I}_i}\right]^{\gamma^{INVPRI^I}_i} \tag{194}$$

$$PK^{PUB}_t = \frac{1}{A^{K\_PUB}} \prod_i \left[\frac{PC_{i,t}}{\gamma^{INVPUB}_i}\right]^{\gamma^{INVPUB}_i} \tag{195}$$

$$IND^F_{k,bus\_f,t} = \phi^F_{k,bus\_f}\left[\frac{R^F_{k,bus\_f,t}}{U^F_{k,bus\_f,t}}\right]^{\sigma^{INV^F}_{k,bus\_f}} KD^F_{k,bus\_f,t} \tag{196}$$

$$IND^I_{k,j,t} = \phi^I_{k,j}\left[\frac{R^I_{k,j,t}}{U^I_{k,j,t}}\right]^{\sigma^{INV^I}_{k,bus\_i}} KD^I_{k,j,t} \tag{197}$$

$$IND_{k,pub,t} = IND^O_{k,pub} pop_t$$

$$VSTK_{i,t} = VSTK^O_{i,t} pop_t$$

$$\begin{aligned} U^F_{k,bus\_f,t} &= PK^{PRI^F}_t\left(\delta^F_{k,bus\_f} + IR_t\right), U^I_{k,j,t} = PK^{PRI^I}_t\left(\delta^I_{k,j} + IR_t\right) \\ \text{and } U_{k,pub,t} &= PK^{PUB}_t\left(\delta^F_{k,pub} + IR_t\right) \end{aligned} \tag{198}$$

9.11 Sets

9.11.1 Industries and commodities

All industries $j,\ jj \in J = \{J_1, ..., J_j, ...\}$
All commodities: $i, ij \in I = \{I_1, ..., I_i, ...\}$
formall sectores $bus\_ft \in BUS\_FT \subset J = \left\{J_1, ..., J_{bus\_ft}, ...\right\}$
Public sectors $pub \in PUB \subset J = \{PUB_1, ..., PUB_{pub}, ...\}$
Private sectors $bus \in BUS \subset J = \{BUS_1, ..., BUS_{bus}, ...\}, BUS \cap PUB = \oslash$
Formal private sectors $bus\_F \in BUS\_F \subset BUS \subset J = \left\{BUS_1, ..., BUS_{bus\_F}, ...\right\}$
Informal private sectors $bus\_I \in BUS\_I \subset BUS \subset J = \left\{BUS_1, ..., BUS_{bus\_I}, ...\right\}, BUS\_I \cap BUS\_F = \oslash$

9.11.2 Production factors

Labor categories: $l \in L = \{L_1, ..., L_l, ...\}$
Formal labor categories: $l\_F \in L\_F \subset L = \left\{L_1, ..., L_{l\_F}, ...\right\}$
Informal labor categories: $l\_I \in L\_I \subset L = \left\{L_1, ..., L_{l\_I}, ...\right\}, L\_I \cap L\_F = \oslash$
Capital categories: $k \in K = \{K_1, ..., K_k, ...\}$
Formal capital categories: $k\_F \in K\_F \subset K = \left\{K_1, ..., K_{k\_F}, ...\right\}$
Informal capital categories: $k\_I \in K\_I \subset K = \left\{K_1, ..., K_{k\_I}, ...\right\}$

9.11.3 Labor supply

labor supply categories: $ls \in LS = \{L_1, ..., L_{ls}, ...\}$

9.11.4 Agents

All agents: $ag, agj \in AG = H \cup F \cup \{GVT, ROW\} = \{H_1, ..., H_h, ..., F_1, ..., F_f, ..., GVT, ROW\}$
Household categories: $h, hj \in H \subset AG = \{H_1, ..., H_h, ...\}$
Firm categories: $f, fj \in F \subset AG = \{F_1, ..., F_f, ...\}$
Non governmental agent:
$agng \in AGNG \subset AG = H \cup F \cup \{ROW\} = \{H_1, ..., H_h, ..., F_1, ..., F_f, ..., ROW\}$

Domestic agents:
$agd \in AGD \subset AG = H \cup F \cup \{GVT\} = \{H_1, ..., H_h, ..., F_1, ..., F_f, ..., GVT\}$

Formal income receiving agents:
$agf \in AGF \subset AG = H \cup \{GVT, F_F, ROW\}$
Informal income receiving agents:
$agi \in AGI \subset AG = H \cup \{F_I\}$
Formal firm
$ff \in FF \subset F \subset AG = \{F_F\}$
Informal firm
$fi \in FI \subset F \subset AG = \{F_I\}$

9.11.5 Periods

Periods: $t \in T = \{T_1, ..., T_t, ...\}$

9.12 Parameters

$aij^F_{i,j}$ = Formal input-output coefficient
$aij^I_{i,j}$ = Informal input-output coefficient
A^{K-PRI^F} = Formal scale parameter (formal private investment funtion)
A^{K-PRI^I} = Informal scale parameter (informal private investment funtion)
A^{K-PUB} = Scale parameter (public investment funtion)
$B^{KD^F}_j$ = Scale parameter (CES - formal composite capital)
$B^{KD^I}_j$ = Scale parameter (CES - informal composite capital)
B^{LDF}_j = Scale parameter (CES - formal composite labor)
$B^{M^F}_i$ = Scale parameter (CES - formal composite commodity)
B^Q_i = Scale parameter (CES - composite commodity)
$B^{VA^F}_j$ = Scale parameter (CES - formal value added)
$B^{VA^I}_j$ = Scale parameter (CES - informal value added)
B^{DD}_i = Scale parameter (CES - composite domestic formal-informal commodity)
$B^{X^F}_{j,i}$ = Scale parameter (CET - formal exports and formal local sales)
$B^{XT^F}_j$ = Scale parameter (CET - formal total output)
$B^{XT^I}_j$ = Scale parameter (CET - informal total output)
$\beta^{KD^F}_{k,j}$ = Share parameter (CES - formal composite capital)
$\beta^{KD^I}_{k,j}$ = Share parameter (CES - informal composite capital)
$\beta^{LD^F}_{l,j}$ = Share parameter (CES - formal composite commodity)
$\beta^{M^F}_i$ = Share parameter (CES - formal composite commodity)
β^Q_i = Share parameter (CES - composite commodity)
β^{DD}_i = Share parameter (CES - composite domestic formal-informal commodity)
$\beta^{VA^F}_j$ = Share parameter (CES - formal value added)
$\beta^{VA^I}_j$ = Share parameter (CES - informal value added)
$\beta^{X^F}_{j,i}$ = Share parameter (CET - formal exports and formal local sales)
$\beta^{XT^F}_j$ = Share parameter (CET - formal total output)
$\beta^{XT^I}_j$ = Share parameter (CET - informal total output)
$\delta^F_{k,j}$ = Depreciation rate of capital k used in formal industry j
$\delta^I_{k,j}$ = Depreciation rate of capital k used in informal industry j
η = Formal price elasticity of indexed transfers and parameters
$frisch(h)$ = Frisch parameter (LES function)

γ_i^{GVT} = Share of commodity i in total current public expenditures on goods and services

$\gamma_i^{INVPRI^F}$ = Share of commodity i in total formal private investment expenditures

$\gamma_i^{INVPRI^I}$ = Share of commodity i in total informal private investment expenditures

γ_i^{INVPUB} = Share of commodity i in total public investment expenditures

$\gamma_{i,h}^{LES}$ = Marginal share of commodity i in household h consumption budget

io_j^F = Coefficient (Leontief - formal intermediate consumption)

io_j^I = Coefficient (Leontief - informal intermediate consumption)

$\lambda_{agf,k}^{RK^F}$ = Share of type k formal capital income received by agent agf

$\lambda_{agi,k}^{RK^I}$ = Share of type k informal capital income received by agent agi

$\lambda_{h,l}^{WL^F}$ =Share of type l formal labor income received by agent household h

$\lambda_{h,l}^{WL^I}$ =Share of type l informal labor income received by agent household h

$\lambda_{ag,agj}^{TR}$ =Share parameter (transfer functions)

$\phi_{k,bus}^F$ = Scale parameter (allocation of formal investment to formal industries)

$\phi_{k,bus}^I$ =Scale parameter (allocation of informal investment to informal industries)

$\rho_j^{KD^F}$ = Elasticity parameter (CES - formal composite capital)

$\rho_j^{KD^I}$ = Elasticity parameter (CES - informal composite capital)

$\rho_j^{LD^F}$ = Elasticity parameter (CES - formal composite labor)

$\rho_j^{M^F}$ =Elasticity parameter (CES - formal composite good)

ρ_j^Q =Elasticity parameter (CES - composite good)

ρ_j^{DD} =Elasticity parameter (CES - composite domestic formal-informal commodity)

$\rho_j^{VA^F}$ = Elasticity parameter (CES - formal value added)

$\rho_j^{VA^I}$ = Elasticity parameter (CES - informal value added)

$\rho_{j,i}^{X^F}$ =Elasticity parameter (CET - exports and local sales)

$\rho_j^{XT^F}$ = Elasticity parameter (CET - formal total output)

$\rho_j^{XT^I}$ = Elasticity parameter (CET - informal total output)

$\sigma_{k,bus\_f}^{INV^F}$ = Elasticity (formal investment demand)

$\sigma_{k,bus\_i}^{INV^I}$ = Elasticity (informal investment demand)

$\sigma_j^{KD^F}$ = Elasticity (CES - formal composite capital)

$\sigma_j^{KD^I}$ = Elasticity (CES - informal composite capital)

$\sigma_j^{LD^F}$ = Elasticity (CES - formal composite labor)

$\sigma_j^{LD^I}$ = Elasticity (CES - informal composite labor)

$\sigma_i^{M^F}$ =Elasticity (CES - formal composite good)

σ_i^Q = Elasticity (CES - composite commodity)
σ_i^{DD} =Elasticity (CES - composite domestic formal-informal commodity)
$\sigma_i^{VA^F}$ =Elasticity (CES - formal value added)
$\sigma_j^{VA^I}$ =Elasticity (CES - informal value added)
$\sigma_{j,i}^{X^F}$ =Elasticity (CET - exports and local sales)
$\sigma_j^{XT^F}$ =Elasticity (CET - formal total output)
$\sigma_j^{XT^I}$ = Elasticity (CET - informal total output)
$\sigma_i^{XD^F}$ = Price elasticity of the world demand for exports of product i
$\sigma_{i,h}^Y$ =Income elasticity of consumption
$tmrg_{ij,i}^F$ = Rate of formal margin i applied to commodity ij
$tmrg_{ij,i}^I$ =Rate of informal margin i applied to commodity ij
$tmrg_{ij,i}^{X^F}$ =Rate of margin i applied to exported commodity x
v_j^F =Coefficient (Leontief - formal value added)
v_j^I =Coefficient (Leontief - informal value added)

9.13 Volume variables

$C_{i,h,t}$ = Consumption of commodity i by type h households
$CG_{i,t}$ = Public consumption of commodity i
$CI_{j,t}^F$ = Formal total intermediate consumption of industry j
$CI_{j,t}^I$ = Informal total intermediate consumption of industry j
$C_{i,h,t}^{MIN}$ = Minimum consumption of commodity i by type h households
$CTH_{h,t}^{REAL}$ =Real consumption expenditures of household h
$DD_{i,t}^F$ = Formal Domestic demand for commodity i produced locally
$DD_{i,t}^I$ = Informal Domestic demand for commodity i produced locally
$DD_{i,t}^T$ = Domestic demand for commodity i produced locally
$DI_{i,j,t}^F$ = Formal Intermediate consumption of commodity i by industry j
$DI_{i,j,t}^I$ = Informal Intermediate consumption of commodity i by industry j
$DIT_{i,t}$ = Total intermediate demand for commodity i
$DS_{h,i}^F$ = Formal supply of commodity i by formal industry j to the domestic market
$DS_{h,i}^I$ = Informal supply of commodity i by informal industry j to the domestic market
$EX_{j,i,t}^F$ = Quantity of product i exported by formal sector j
$EXD_{i,t}^F$ = World demand for exports of product i
G_t^{REAL} =Real current government expenditures on goods and services
$GDP_t^{BP^F\_REAL}$ = Formal real GDP at basic prices
$GDP_t^{BP^I\_REAL}$ = Informal real GDP at basic prices
$GDP_t^{MP^F\_REAL}$ = Formal Real GDP at market prices
$GFCF_t^{PRI^F\_REAL}$ =Real formal private gross fixed capital formation
$GFCF_t^{PRI^I\_REAL}$ =Real informal private gross fixed capital formation

$IM^F_{i,t}$ = Quantity of product i imported

$IND^F_{k,bus,t}$ =Volume of new type k capital investment to formal industry j

$IND^I_{k,bus,t}$ =Volume of new type k capital investment to informal industry j

$INV_{i,t}$ = Final demand of commodity i for investment purposes

$INV^{PRI^F}_{i,t}$ = Final demand of commodity i for formal private investment purposes

$INV^{PRI^I}_{i,t}$ =Final demand of commodity i for informal private investment purposes

$INV^{PUB}_{i,t}$ = Final demand of commodity i for public investment purposes

$KD^F_{k,j,t}$ = Demand for type k capital by formal industry j

$KD^I_{k,j,t}$ = Demand for type k capital by informal industry j

$KDC^F_{j,t}$ = Formal Industry j demand for composite capital

$KDC^I_{j,t}$ = Informal Industry j demand for composite capital

$KS^F_{k,t}$ = Formal supply of type k capital

$KS^I_{k,t}$ = Informal supply of type k capital

$LD^F_{l,j,t}$ = Formal Industry j demand for type l formal labor

$LD^I_{l,j,t}$ = Informal Industry j demand for tyme l informal labor

$LDC^F_{j,t}$ = Formal Industry j demand for composite formal labor

$LDC^I_{j,t}$ = Inormal Industry j demand for composite formal labor

$LS_{ls,t}$ = Supply of type ls labor

$MRGN_{i,t}$ = Demand for commodity i as a trade or transport margin

$Q_{i,t}$ = Quantity demanded of composite commodity i

$Q^F_{i,t}$ = Quantity demanded of formal composite commodity i

$VA^F_{j,t}$ = Value added of formal industry j

$VA^I_{j,t}$ =Value added of informal industry j

$VSTK_{i,t}$ = Inventory change of commodity i

$XS^F_{j,i,t}$ = Formal Industry j production of commodity i

$XS^I_{j,i,t}$ = Informal Industry j production of commodity i

$XST^F_{j,t}$ = Total aggregate output of formal industry j

$XST^I_{j,t}$ =Total aggregate output of informal industry j

9.14 Price variables

e_t =Exchange rate (price of foreign currency in local currency)

IR_t =Interest rate

$P^F_{j,i,t}$ =Basic price of formal industry j's production of commodity i

$P^I_{j,i,t}$ =Basic price of informal industry j's production of commodity i

$PC_{i,t}$ =Purchaser price of composite commodity i (including all taxes and margins)

$PQ^F_{i,t}$ =Price of formal composite commodity i

$PCI^F_{j,t}$ =Intermediate consumption price index of formal industry j

$PCI^I_{j,t}$ =Intermediate consumption price index of informal industry j

$PD^F_{i,t}$ =Formal Price of local product i sold on the domestic market (including all taxes and margins)

$PD^I_{i,t}$ =Informal Price of local product i sold on the domestic market (including all margins)

$PE^F_{i,t}$ =Formal price received for exported commodity i (excluding export taxes)

$PE^{FOB}_{i,t}$ = FOB price of exported commodity i (in local currency)

$PIXCON_t$ =Consumer price index

$PIXGDP^F_t$ =Formal GDP deflator

$PIXGDP^I_t$ = Informal GDP deflator

$PIXGVT_t$ =Public expenditures price index

$PIXINV^{PRI^F}_t$ = Formal private investment price index

$PIXINV^{PRI^I}_t$ = Informal private investment price index

$PIXINV^{PUB}_t$ = Public investment price index

$PK^{PRI^F}_t$ =Formal Price of new private capital

$PK^{PRI^I}_t$ =Informal Price of new private capital

PK^{PUB}_t =Price of new public capital

$PL^F_{i,t}$ =Formal price of local product i (excluding all taxes on products)

$PL^I_{i,t}$ = Informal price of local product i

$PM^F_{i,t}$ =Formal Price of imported product i (including all taxes and tariffs)

$PP^F_{j,t}$ = Formal Industry j unit cost including taxes directly related to the use of capital and labor but excluding other taxes on production

$PP^I_{j,t}$ =Infomal Industry j unit cost

$PT^F_{j,t}$ =Basic price of Formal industry j's output

$PT^I_{j,t}$ = Basic price of Informal industry j's output

$PVA^F_{j,t}$ =Price of formal industry j value added (including taxes on production directly related to the use of capital and labor)

$PVA^I_{j,t}$ =Price of informal industry j value added

$PWM_{i,t}$ =World price of imported product i (expressed in foreign currency)

$PWX_{i,t}$ =World price of exported product i (expressed in foreign currency)

$R^F_{k,j,t}$ = Rental rate of type k capital in formal industry j

$R^I_{k,j,t}$ = Rental rate of type k capital in informal industry j

$RC^F_{j,t}$ = Rental rate of formal industry j composite capital

$RC^I_{j,t}$ =Rental rate of informal industry j composite capital

$RTI^F_{k,j,t}$ =Rental rate paid by formal industry j for type k capital including capital taxes

$U^F_{k,bus\_f,t}$ =User cost of type k capital in formal industry j

$U^F_{k,PUB,t}$ =User cost of type k capital in public sector

$U^I_{k,j,t}$ =User cost of type k capital in informal industry j

$W^F_{l,j,t}$ =Wage rate of type l formal labor

$W^I_{l,t}$ =Wage rate of type l informal labor

$WC^I_{j,t}$ =Wage rate of informal industry j composite labor

$CT_{j,t}$ =Wage rate of formal industry j composite labor

$WTI^F_{l,j,t}$ =Wage rate paid by formal industry j for type l labor including payroll taxes

9.15 Nominal (value variables)

CAB_t = Current account balance

$CTH_{h,t}$ =consumption expenditures of household h

G_t = current government expenditures on goods and services

$GDP^{BP^F}_t$ = Formal real GDP at basic prices

$GDP^{BP^I}_t$ = Informal real GDP at basic prices

$GDP^{MP^F}_t$ = Formal Real GDP at market prices

$GDP^{FD^I}_t$ = Informal real GDP at basic prices

$GDP^{IB^F}_t$ = Formal Real GDP at market prices

$GFCF_t$ = Gross fixed capital formation

IT_t =Total investment expenditures

$IT^{PRI^F}_t$ =Total formal investment expenditures

$IT^{PRI^I}_t$ =Total informal investment expenditures

IT^{PUB}_t =Total public investment expenditures

$RKDPO$ =Type k capital income in industry j

$RKDPO\_F$ = formal Type k capital income in industry j

$RKDPO\_I$ = Informal Type k capital income in industry j

$SF^F_{FF,t}$ = Savings of formal businesses

$SF_{FI,t}$ = Savings of informal businesses

SG_t = Government savings

$SH_{h,t}$ = Savings of type h households

$SROW_t$ =Rest-of-the-world savings

$TDF_{FF,t}$ =Income taxes of formal businesses

$TDFT_t$ =Total government revenue from business income taxes

$TDH_{h,t}$ =Income taxes of type h households

$TDFT_t$ =Total government revenue from household income taxes

$TICT_t$ =Total government receipts of indirect taxes on commodities

$TIC_{i,t}$ =Government revenue from indirect taxes on product i

$TIKT_t$ =Total government revenue from from taxes on capital

$TIK^F_{k,j,t}$ = Government revenue from taxes on type k capital used by formal industry j

$TIMT_t$ =Total government revenue from import duties

$TIM_{i,t}$ =Government revenue from import duties on product i

$TIPT_t$ =Total government revenue from production taxes (excluding taxes directly related to the use of capital and labor)

$TIP^F_{j,t}$ = Government revenue from taxes on formal industry j production (excluding taxes directly related to the use of capital and labor)

$TIWT_t$ =otal government revenue from payroll taxes

$TIW^F_{l,j,t}$ = Government revenue from payroll taxes on formal type l labor in formal industry j

$TIXT_t$ =Total government revenue from export taxes

$TIX_{i,t}$ =Government revenue from export taxes on product i
$TPRCTS_t$ =Total government revenue from taxes on products and imports
$TPRODN_t$ = Total government revenue from other taxes on production
$TR_{ag,agj,t}$ =Transfers from agent agj to agent ag
$YDF^F_{FF,t}$ =Disposable income of type f formal businesses
$YDF^F_{FI,t}$ =Disposable income of type f informal businesses
$YDH_{h,t}$ = Disposable income of type h households
$YF^F_{FF,t}$ = Total income of type f formal businesses
$YF^I_{FI,t}$ = Total income of type f informal businesses
$YFK^F_{FF,t}$ = Capital income of type f formal businesses
$YFK^I_{FI,t}$ =Capital income of type f informal businesses
$YFTR^F_{FF,t}$ =Transfer income of type f formal businesses
$YFTR^I_{FI,t}$ = Transfer income of type f informal businesses
YG_t =Total government income
YGK_t = Government capital income
$YGTR_t$ =Government transfer income
$YH_{h,t}$ = Total income of type h households
$YHK_{h,t}$ =Capital income of type h households
$YHK^F_{h,t}$ =Formal Capital income of type h households
$YHK^I_{h,t}$ =Informal Capital income of type h households
$YHL_{h,t}$ =Labor income of type h households
$YHTR_{h,t}$ = Transfer income of type h households
$YHTR_{h,t}$ =Formal Transfer income of type h households
$YHTR_{h,t}$ =Informal Transfer income of type h households
$YROW_t$ =Rest-of-the-world income

9.16 Rates and intercepts

$sh0_{h,t}$ = Intercept (type h household savings)
$sh1_{h,t}$ =Slope (type h household savings)
$tr0_{h,t}$ =Intercept (transfers by type h households to government)
$tr1_{h,t}$ = Marginal rate of transfers by type h households to government
$ttdf0_{FF,t}$ = Intercept (income taxes of type f formal businesses)
$ttdf1_{FF,t}$ = Marginal income tax rate of type f informal businesses
$ttdh0_{h,t}$ =Intercept (income taxes of type h households)
$ttdh1_{h,t}$ =Marginal income tax rate of type h households
$ttic_{i,t}$ =Tax rate on commodity i
$ttik^F_{k,j,t}$ = Tax rate on type k capital used in formal industry j
$ttim_{i,t}$ =Rate of taxes and duties on imports of commodity i
$ttip^F_{j,t}$ =Tax rate on the production of industry j
$ttiw^F_{l,j,t}$ =Tax rate on type l worker compensation in industry j
$ttix_{i,t}$ = Export tax rate on exported commodity i

Agradecimientos

Esta serie de documentos de trabajo es financiada por el programa "Inclusión productiva y social: programas y políticas para la promoción de una economía formal", código 60185, que conforma Colombia Científica-Alianza EFI, bajo el Contrato de Recuperación Contingente No.FP44842-220-2018.

Acknowledgments

This working paper series is funded by the Colombia Científica-Alianza EFI Research Program, with code 60185 and contract number FP44842-220-2018, funded by The World Bank through the call Scientific Ecosystems, managed by the Colombian Ministry of Science, Technology and Innovation.